Development and analysis of an improved finite volume method for polyhedral meshes

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Abstract: The objectives of the present study are to analyze the accuracy of the finite volume method for a polyhedral mesh and to derive a new method for a improved accuracy and stability. By comparing the error terms from grids with different shapes, it is shown that a polyhedral grid has a better accuracy than a triangular/tetrahedral grid. An improved finite volume method is derived by minimizing the error terms from interpolating face variables. Different methods for error minimization are compared and analyzed. The advantages of the proposed method are shown by examples of laminar and turbulent flows.

Keywords: Unstructured Polyhedral Mesh, Finite Volume Method, Skewness Correction, Low dissipative scheme.

1 Introduction

It is well known that the quality of a computational mesh significantly affects the size of the error from a flow simulation. Among different shapes of a mesh element, there have been recent interests in unstructured polyhedral elements - polygonal grids with more faces than a hexahedral grid in 3-D. Advantages of polyhedral grids compared to the other types of unstructured grids were already mentioned in several previous studies (e.g., [1, 2]). First, a polyhedral grid provides with more accurate discretization of derivative operators using a relatively large number of neighboring points. Second, it is less susceptible to degradation of the error by skewness and stretching of meshes. Another possible advantage for a problem with fluid-structure interaction is relatively easier deformation like merging or splitting of cells. In literature, there are several previous studies on the finite volume method for polyhedral meshes. However, there are only few studies on detailed analysis of the error term from finite volume discretization for a polyhedral mesh.

There are two objectives of the present study. The first one is to prove the advantage of a polyhedral mesh in terms of the error size by analytic description of the error term from the finite volume method. The second one is to derive an improved finite volume discretization for a polyhedral mesh with optimized accuracy and stability. The features and usefulness of the present method are shown by examples of laminar and turbulent flows.

2 Problem Statement and Contributions

In the present study, the variable density formulation of the Navier-Stokes equations is considered:

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right], \quad \frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0. \quad (1)$$

All the spatial terms in these equations are conservative and are composed of the divergence and gradient operators. For polyhedral grids, the finite volume descriptions of these operators for a vector u_i and a scalar ϕ are

$$\frac{\partial u_i}{\partial x_i}\Delta V = \sum_f u_i^f A_i^f, \quad \frac{\partial \phi}{\partial x_i}\Delta V = \sum_f \phi^f A_i^f, \quad (2)$$

where the index f denotes a face of the polyhedral element and A_i^f is the outward-normal area vector of a face. Important questions regarding Eq. (2) are i) how to compute the face variables ϕ^f and u_i^f, ii) how to analyze and establish the accuracy and iii) how to enhance stability of the discretization method. Contributions of the present study related to these aspects are:

- By comparing grids with different numbers of faces using an analytic form of the error term, it is shown that a polyhedral grid has a better accuracy than a triangular/tetrahedral grid.
- An improved finite volume method is derived for a polyhedral mesh by minimizing the error terms from interpolating the face variables. Different methods for error minimization are compared and analyzed.
- Improved accuracy by the proposed method is shown by test cases of laminar and turbulent flows. A method to stabilize a simulation with a reduced amount of dissipation is discussed.

References

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