A discrete-forcing immersed boundary method for a thin flexible body

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Abstract: In the present study, an immersed boundary (IB) method for the simulation of flow around a thin flexible body is presented. The present method is based on a discrete-forcing immersed boundary method proposed by Kim et al. (JCP, 2001). The dynamic equation for a thin flexible body is coupled with the incompressible Navier-Stokes equations. With the proposed method, we simulate several flow problems including flows around a flexible filament and a hovering insect wing at CFL=0.6 – 1.0.

Keywords: Immersed boundary method, Discrete-forcing, Thin-flexible body, Fluid-structure interaction.

1 Introduction

A thin flexible body interacting with ambient fluid is observed in nature and engineering applications such as a flapping flag, hovering insect wing, etc. These thin flexible bodies deform due to the tension and bending forces as well as the hydrodynamic force. Thus, the fluid-structure interaction is an important phenomenon in simulation of this flow. The IB method is an efficient and effective tool because it uses Cartesian mesh and does not require mesh regeneration in time. A few different versions of IB method [1-3] have been developed using the continuous forcing approach [4]. In this study, we develop a discrete-forcing IB method for the simulation of flow around a thin flexible body. The present method is based on the discrete-forcing IB method for a stationary body [5]. The use of discrete forcing approach is expected to reduce the severe computational time step restriction which occurs with the continuous forcing approach [1-3]. The dynamic equation for a thin flexible body is coupled with incompressible Navier-Stokes equations through the hydrodynamic force.

2 Numerical details

The governing equations are solved in the Eulerian coordinates using an IB method [5]:

$$\frac{\partial u_i}{\partial t}+\frac{\partial u_i u_j}{\partial x_j}=-\frac{\partial p}{\partial x_i}+\frac{1}{\mathrm{Re}}\frac{\partial^2 u_i}{\partial x_j \partial x_j}+f_i \tag{1}$$

$$\frac{\partial u_i}{\partial x_i}-q=0 \tag{2}$$

where x_i is the Cartesian coordinates, u_i the corresponding velocity, p the pressure, f_i the momentum forcing, and q the mass source/sink. We use a semi-implicit fractional step method: Crank-Nicolson method for the viscous term and third-order Runge-Kutta method for the convection term. The second-order central difference scheme is used for all the terms. The thin flexible body is described in the Lagrangian coordinates, and is segmented by finite number of thin blocks. Each block is moved by the tension, bending, buoyancy and hydrodynamic forces:

$$\rho\frac{\partial^2 X_i}{\partial t^2}=\frac{\partial}{\partial s_m}\left\{K_{mn}^T\left(\frac{\partial X_i}{\partial s_m}\cdot\frac{\partial X_i}{\partial s_n}-\frac{\partial X_i^0}{\partial s_m}\cdot\frac{\partial X_i^0}{\partial s_n}\right)\frac{\partial X_i}{\partial s_n}\right\}-\frac{\partial^2}{\partial s_m \partial s_n}\left(K_{mn}^B\frac{\partial^2 X_i}{\partial s_m \partial s_n}\right)-(\rho-1)\frac{g_i}{g}Fr+\frac{F_i}{V}, \tag{3}$$

where ρ is the density ratio, X_i the central position of each block, V the volume of each block, $K^T{}_{mn}$ the tension coefficients, $K^B{}_{mn}$ the bending coefficients, Fr the Froude number, and F_i the hydrodynamic force. The hydrodynamic force is obtained directly from the integration of Eq. (1).

3 Results

With the proposed IB method, we simulate several flow problems. The result from flow around a flexible filament with fixed leading edge agrees very well with those from previous study [6]. A snap shot of flow field is given in Fig. 1(a). We also simulate the flow over a flexible 2D wing in normal hovering, where the bending rigidity of wing is obtained from the result of Combes and Daniel [7]. Figure 1(b) shows the vorticity contours around a flexible wing. Shape deformation due to the elastic property is clearly seen. Figure 1(c) shows the vortical structures from a flapping flag together with the shape of 3D flag. For all the computations considered in this study, the maximum CFL numbers are 0.6 – 1.0, showing the efficiency of the present discrete forcing approach as opposed to that of the continuous forcing approach.

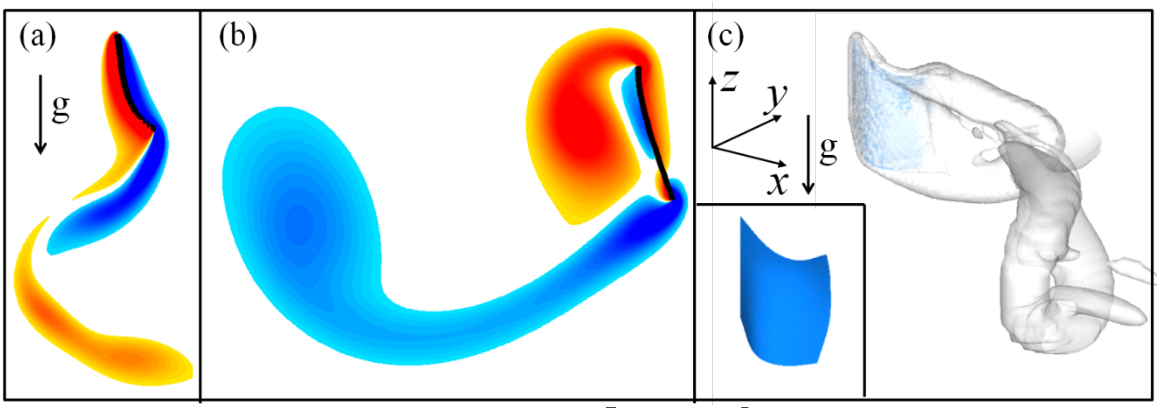

Figure 1: (a) Flexible filament (Re = 200, ρ = 151, K^T = 2500, K^B = 0.15, Fr = 0.5); (b) flexible wing (Re = 75, ρ = 1000, K^T = 30000, K^B = 200); (c) 3D flag (Re = 200, ρ = 101, $K_{11}{}^T = K_{22}{}^T$ = 2500, $K_{12}{}^T = K_{21}{}^T$ = 100, $K_{mn}{}^B$ = 0.01, Fr = 0.1).

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