Convergence Acceleration of High Order Numerical Simulations using a Hybrid Spectral Difference / Finite Volume Multigrid Method

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Abstract: The goal of this paper is to show how numerical simulations of fluid flow using high order methods for unstructured meshes can be speeded up using a hybrid multigrid method. In our work we accelerate the steady state convergence of a Spectral Difference code by coupling it to a Finite Volume solver. While we want to obtain the solution for the SD code, low frequency corrections to the solution are computed using the finite volume code.

Keywords: Numerical Algorithms, Computational Fluid Dynamics, Spectral Differences, Multigrid.

1 Introduction

High order numerical methods for unstructured meshes have recently received a lot of attention in the CFD community. However, these methods can be slow to converge due to the very strong limitations on the time steps that can be taken. While a lot of work is devoted in the community to developing implicit schemes to overcome this difficulty, we focus instead on a multigrid approach to solving the problem.

2 Hybrid multigrid method

The idea behind classical multigrid methods is to compute the solution on a fine mesh and corrections to the solution on successive coarser meshes to eliminate the various components of the error at a faster rate. The method relies on the fact that low frequency components of the error can be represented on coarser meshes and be damped or propagated outside of the numerical domain more effeciently on these meshes. In our work, the solution is computed on an unstructured mesh using the Spectral Difference method and low frequency corrections to the solution are performed on multiple levels using a Finite volume code. The reason behind this idea is that convergence acceleration using Finite Volume multigrid method is well known and a easier to implement than a full SD multigrid method.

2.1 SD and FV flow solvers

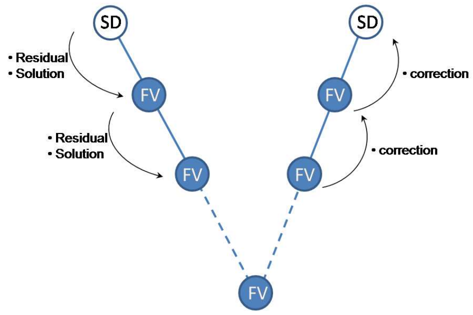
The final paper will describe briefly our Spectral Difference flow solver as well as the Finite Volume solver (based on FLO103).

2.2 The multigrid Cycle

We use a hybrid multigrid method. While the goal is to obtain the solution using the SD method, corrections are computed using a Finite volume code.

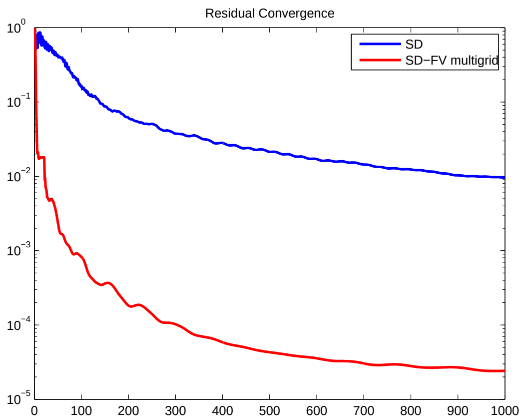
2.3 Transfers between SD and FV meshes

Describe bilinear interpolations and the various steps between SD to FV (transfer solution and residual) and FV to SD (transfer correction).

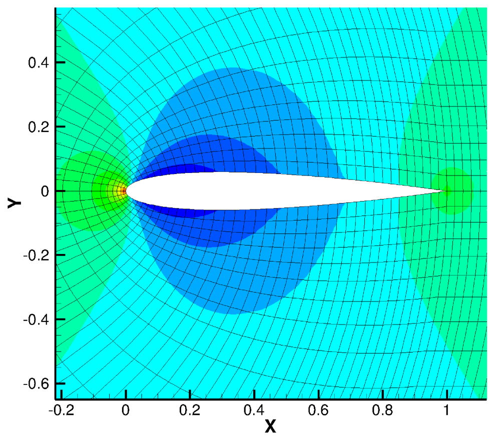


3 Results

The method was used to compute the steady state inviscid subsonic flow around a NACA 0012 airfoil. The solution is expected to be of order 4 on a mesh counting 2016 cells. The various finite volume meshes used in the multigrid cycles count 512×64 (32,768), 256×32 (8,192), 128×16 (2,048), 64×4 (256) and 32×2 (64) cells. Therefore, the multigrid counts a total of 6 levels in this example ($5 + 1$ for the SD mesh). Figure 1 shows a considerable acceleration in convergence of the residual.



(a) Residual Convergence



(b) Density

Figure 1: Multigrid acceleration of the flow around an airfoil

4 Conclusion and Future Work

Considerable speed up of the SD method was obtained and preliminary results are very promising. Future work includes better tuning of the method and comparisons with other acceleration techniques, in particular the use of implicit schemes.