Convergence Error Estimation and Convergence Acceleration in Iteratively Solved Problems

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Abstract: Two new methods are developed for convergence error estimation and convergence acceleration in iteratively solved problems. The convergence error estimation method is based on the eigenvalue analysis of linear systems, but it can also be used for nonlinear systems. The convergence of iterative method is accelerated by subtracting convergence error from the iteratively calculated solutions. The performances of these methods are demonstrated for the Navier-Stokes equations.

Keywords: Convergence Error, Convergence Acceleration, Iteratively Solved Problems

1 Introduction

There is a great interest in estimating the convergence error. Knowing when to stop iteration is important in terms of computational efficiency and accuracy. In most of the iteratively solved problems, the reduction in residual is used as a stopping criterion. Unfortunately, the reduction in residual may not be a reliable measure for the convergence error. Different methods were developed to estimate the convergence error. Ferziger and Peric [1,2] used eigenvalue analysis of linear systems in convergence error estimations. In this study, the convergence error vector is expressed as the linear combination of the correction vectors [3]. The coefficients of the correction vectors are calculated using the least-squares minimization. Once knowing how to estimate the convergence error, the next step is to develop a convergence acceleration method. In literature, there is almost no research on convergence acceleration based on the convergence error estimation. The convergence acceleration method presented in this study is based on the estimation of the exact solution.

2 Problem Statement

Using the eigenvalue analysis of linear systems, the following method is developed [3]. In this method, the convergence error vector, ε, at iteration *n+1*, is calculated as:

$$\varepsilon^{n+1} = \frac{\left(\sum_{m=1}^{2M_{eigen}} C_m\right)\delta^n + \left(\sum_{m=2}^{2M_{eigen}} C_m\right)\delta^{n-1} + \left(\sum_{m=3}^{2M_{eigen}} C_m\right)\delta^{n-2} + \ldots + \left(C_{2M}\right)\delta^{n-2M_{eigen}+1}}{\sum_{m=1}^{2M_{eigen}} C_m - 1} \quad (1)$$

In the equation above, M_{eigen} is the number of eigenvalues. The coefficients C_m's are real numbers and they are determined from the least squares solution of the following equation.

$$\delta^n = \sum_{m=1}^{2M_{eigen}} C_m \delta^{n-m} \quad (2)$$

In the calculation of these coefficients, the correction vectors, δ, from the present and last $2M_{eigen}$ iterations must be stored. Although, increasing the number of eigenvalues may improve the accuracy of convergence error estimation, this improvement may also cause an increase in the memory requirement to store correction vectors from previous iterations.

Once the convergence error is determined, the exact solution, w, which exactly satisfies the discretized governing equations, can be estimated. The convergence can be accelerated by subtracting convergence error from the iterative solution, $\tilde{w}^n$.

$$w = \tilde{w}^n - \varepsilon^n \tag{3}$$

The performances of the convergence error estimation and convergence acceleration methods are tested in the solution of Navier-Stokes equations. The flow around the NACA0012 airfoil is solved at a transonic flow condition (M=0.730, α=2.78^o, Re=6.5x10^6). In the solution of equations, four-stage Runga-Kutta scheme, local time stepping and four level multigrid method are implemented. Figure 1 shows the accuracy of the convergence error estimation method for a CFL number of 1.5. The results show that the proposed method can accurately estimate the convergence error, however, the residual may not be a good parameter to predict the convergence error. Figure 2 shows the performance of the convergence acceleration method with a maximum CFL number of 1.9. If no convergence acceleration method is used, the norm value of residual is reduced to the order of machine epsilon in 20,000 iterations. If 16 eigenvalues are used in the convergence acceleration method, the residual can be reduced to the same level in 3000 iterations. Similarly if the number of eigenvalues is increased to 256, the same convergence level can be achieved in 2500 iterations.

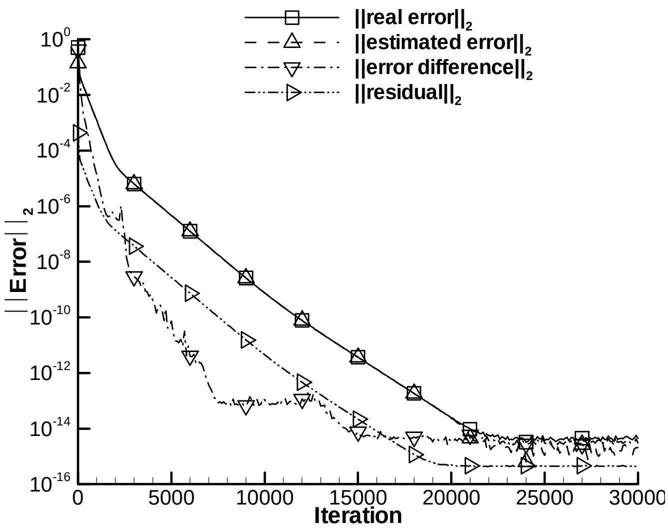

Figure 1. Convergence error estimation

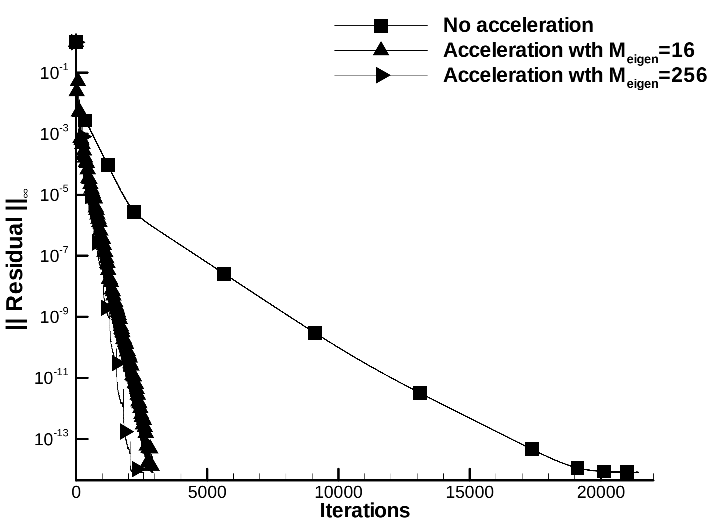

Figure 2. Convergence acceleration

3 Conclusion and Future Work

Two new methods are developed. In the first method, the convergence error is estimated in iteratively solved problems. The method is based on the eigenvalue analysis of linear systems. In the second method, the convergence of an iterative method is accelerated by estimating the exact solution. The performances of these methods are demonstrated in the solution of Navier-Stokes equations. The results show that the convergence error can be accurately estimated with the developed method. The residual itself, on the other hand, is not considered to be a reliable parameter to predict the convergence error. The proposed convergence acceleration method reduces the number of iterations. More results will be included in the full paper.

References

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