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LES modeling with high-order flux reconstruction and spectral difference schemes

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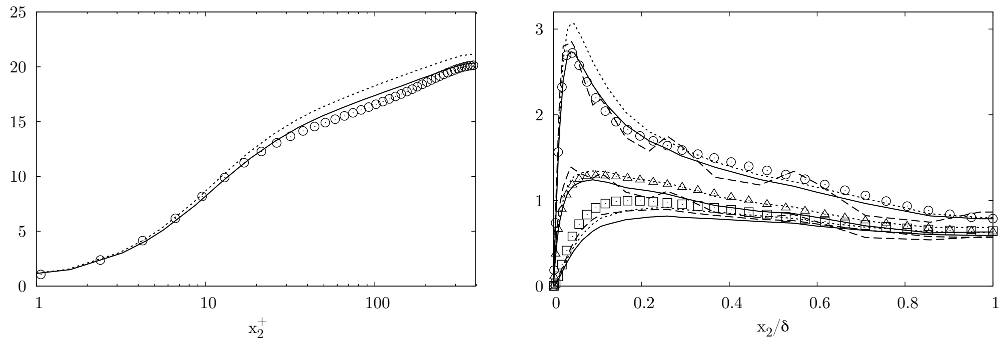
1 Introduction

Notwithstanding the considerable effort which has been devoted to the development of accurate and relatively reliable Sub-Grid Scale (SGS) models for Large-Eddy Simulation (LES), the underlying numerical methods, which are available within the framework of industrial computational fluid dynamics applications generally rely upon highly dissipative schemes. The inherent numerical dissipation introduced by such numerical schemes limits their ability to correctly represent the whole spectrum resolved in LES. Hence it is necessary to combine high order numerical schemes with advanced SGS modeling techniques in order for LES to become a valuable and reliable tool for fundamental flow physics and industrial applications. Unfortunately, most of the available high-order numerical schemes are designed to be used on cartesian or very smooth structured curvilinear meshes and therefore they are inadequate to simulate turbulent flows over complex geometries. In the current work, a high-order unstructured solver is combined with an explicit filtering LES method, thus allowing highly accurate turbulent flow computations on realistic geometries that were previously only possible with low-order schemes.

High-order numerical schemes for solving the compressible Navier-Stokes equations on unstructured grids have been widely studied during the last decade. By far the most mature and widely used of these schemes are based on the Discontinuous Galerkin (DG) method [2, 6]. However, several alternative high-order methods have been recently proposed, including Spectral Difference (SD) type schemes [8, 9, 13], which potentially offer increased efficiency compared with DG methods (as well as being simpler to implement). In the context of the SD method for three-dimensional unstructured hexahedral grids, the present study addresses the implementation of a structural SGS model based on the scale similarity assumption, namely, the WALE Similarity Mixed model (WSM) proposed by Lodato et al. [10]. The proposed implementation of a constrained discrete filter of arbitrary order for the SD method will be suitable for a broad class of numerical schemes based on the Discontinuous Finite Element (DFE) representation of the solution, such as the family of energy stable schemes that can be obtained within the unifying Flux Reconstruction (FR) framework [1, 3–5, 14–16].

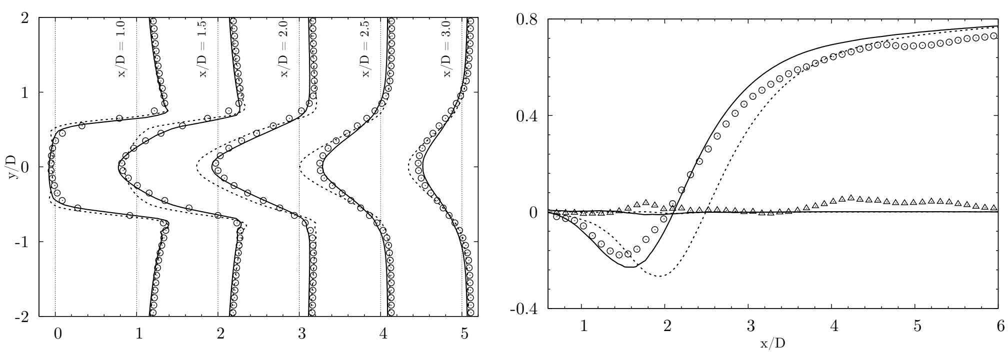
2 Main results

In view of the application of SGS modeling approaches that make use of explicit spatial filtering, a new class of Constrained Discrete (CD) filters have been developed in order to ensure an almost uniform cutoff frequency within the element and a good numerical behavior. These filters can be applied to arbitrary distributed points and are completely local inside the standard element, therefore they are relatively easy to implement and do not involve any additional complication for the parallelism of the numerical solver. Compared with the Restriction-Prolongation (RP) filter [12], the developed filters are numerically better behaved, as it can be observed in Figure 1, where the results obtained on the $Re_\tau = 395$ channel flow with the SD scheme and the WSM model [10] are shown. Further validation on the turbulent flow past a confined circular cylinder at Reynolds number 2 580 is also reported (cf. Figure 2), and results from a relatively coarse mesh with with 498 690 degrees of freedom are compared to PIV measurements [7]. Overall, the performance of the actual WSM model implementation in conjunction with the SD method and the new discrete filters is extremely satisfactory. In both the flow configurations studied, statistical moments are generally improved when the SGS model is used. The new discrete filters eliminate completely spurious numerical artifacts that were observed when using existing filtering operators for DFE type schemes.



(a) (b)

Figure 1: Mean streamwise velocity profile (a) and RMS of velocity fluctuations (b) for turbulent channel flow at $Re_\tau = 395$ ($N = 4$): ——— , WSM model (CD filter); - - - - - , WSM model (RP filter); ········ , no SGS model; symbols, DNS data [11]. ○ , U^+ or u^+_{rms}; □ , v^+_{rms}; △ , w^+_{rms}.



(a) (b)

Figure 2: Mean velocity profiles at different locations downstream of the cylinder (a) and along its wake (b): ——— , WSM model; ········ , no SGS model; symbols (○ , streamwise; △ , vertical), experiments.

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