Improved Flux Formulations for Unsteady Low Mach Number Flows

Abstract for Seventh International Conference on Computational Fluid Dynamics in Big Island, Hawai'i 2012

A. Hosangadi*, J. Sachdev*, and V. Sankaran**
Corresponding author: hosangad@craft-tech.com

* Combustion Research and Flow Technology, Inc., 6210 Keller's Church Rd., Pipersville, PA 18947 (CRAFT Tech), USA
** Air Force Research Laboratory (AFRL), Space and Missile Propulsion Division, Edwards AFB USA

1. Abstract

Preconditioning techniques are being used very effectively for *steady* low Mach number flows in time-marching algorithms by scaling the acoustic speed to the local convective velocities and thereby providing the correct dissipation levels to both the pressure and the scalar velocity/temperature fields [1]-[4]. However for unsteady low Mach numbers flows these more simplistic scaling rules become inadequate since the unsteady time scales associated with acoustic wave propagation are very different from those for velocity propagation; at high local cell Strouhal numbers resolving the acoustic time scales become important and the steady preconditioning matrix that effectively filters the acoustics from the solution becomes far too dissipative in resolving the pressure wave propagation. As a consequence steady Mach number scaling parameters, do not maintain accuracy uniformly across both pressure and velocity/energy transport, and furthermore also place conflicting requirements on efficient convergence at each time level.

The development of multi-scale dissipation that can independently tailor dissipation for the pressure propagation and the convective scalar is the focus of our paper. Both AUSM and coupled flux-differenced family of schemes are considered. For flux differenced schemes, generalized "blending" methodologies have been developed (extending earlier work by Potsdam et al. [2]) wherein "unsteady" preconditioning is used for the pressure wave propagation, while "steady" preconditioning is used for the convected scalars that propagate at the fluid velocity. Specifically, two different "blending" formulations, that take advantage of the analytical algebraic form of the upwind dissipation matrix in formulating the unsteady and steady terms, will be evaluated in the present article.

A second approach is based on the AUSM family of schemes because the scalar flux formulation lends itself naturally to independent dissipation formulations for the pressure and scalar equations. Two forms of the AUSM scheme were initially analyzed: the AUSM+up by Liou [3] and the SLAU scheme by Shima and Kitamura [4]. The AUSM+up scheme was shown to be tailored for steady low Mach number flows, while the SLAU scheme dissipation was shown to be more appropriate for unsteady and transonic flows. Based on this study, a more generalized AUSM-unsteady scheme has been formulated using an unsteady Mach number parameter that provides a unified framework for steady and unsteady flows at all Mach numbers. Furthermore, a new pressure dissipation term is developed for low Mach number acoustic flows to suppress spurious oscillations for flows where pressure differences drive the fluid velocity.

Both the "blended" flux difference and AUSM-unsteady formulation were tested rigorously for the following three test cases that include both hydrodynamic and acoustic instabilities: 1) Unsteady inviscid Lamb vortex problem (hydrodynamic instability), 2) Unsteady inviscid flow in a pipe with fluctuating back pressure (mixed acoustic and hydrodynamic instability), and 3) Shock tube with small pressure differences (pure acoustic problem). In general, both schemes provide accurate unsteady results over a

"Approved for public release; distribution unlimited" (PA Clearance Number 111054)

wide range of Mach numbers with good inner iteration convergence. However, there are two notable exceptions. For multi-dimensional flows such as the vortex propagation problem, the pressure field in the "blended" flux-difference formulations displays spatial distortions while the AUSM-unsteady formulation preserves the correct solution as shown in Figure 1.

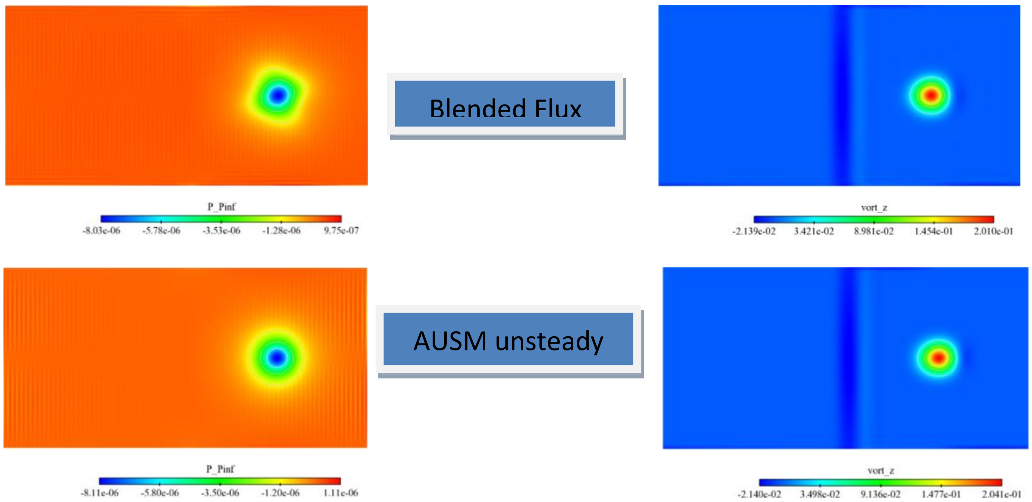
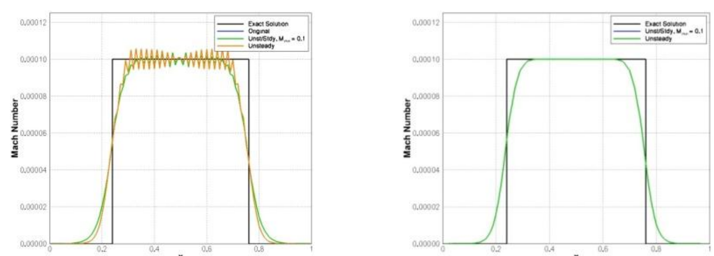


Figure 1: Pressure and vorticity field for Vortex Propagation Problem

The second problem is related to the pure acoustic test case of a very weak amplitude shock tube. Both the blended flux difference and AUSM-unsteady schemes show oscillations in the pressure and velocity field as shown in Figure 2(a). For the AUSM-unsteady schemes, these spurious oscillations can be eliminated by adding a new dissipation term to the pressure flux as shown in Figure 2(b). We have hitherto not devised an equivalent procedure for the blended scheme, although we observe that reverting to the baseline unsteady preconditioning procedure is effective in eliminating the oscillations. In conclusion, the new flux formulations presented here achieve our primary goals for simulating unsteady low Mach number flows: 1) Flux procedures that work accurately and efficiently for both hydrodynamic and acoustic instabilities, and 2) Solution accuracy and convergence that are not sensitive to choice of time step and unsteady preconditioning parameter.



(a) (b)

Figure 2: Solutions for low amplitude shock tube with and without pressure dissipation term for the AUSM unsteady formulation.

2. References

[1] Venkateswaran, S., and Merkle C.L., "Analysis of Preconditioning Methods for the Euler and Navier Stokes Equation", VKI Lecture Series Monographs on Computational Fluid Dynamics, VKI LS 1999-03, von Karman Institute, March, 1999.

[2] Potsdam, M. A., Sankaran, V., and Pandya, S. A, "Unsteady Low Mach Preconditioning with Application to Rotorcraft Flows," Paper No. AIAA-2007-4473, 18th AIAA Computational Fluid Dynamics Conference, Miami, FL, 25-28 Jun 2007.

[3] Liou, M., "A Sequel to AUSM, Part II, AUSM+up for All Speeds," *Journal of Computational Physics*, Vol. 214, No. 1, pp. 127-170, 2006.

[4] Shima, E, and Kitamura, K., "CFD Method for Aero-Acoustics using all-speed numerical flux and preconditioned implicit time integration," 20th AIAA Computational Fluid Dynamics Conference, Honolulu, HI, 27-30 Jun 2011.