Flux Functions for Reducing Numerical Shockwave Anomalies

Daniel W. Zaide and Philip L. Roe
Corresponding Author: zaidedan@umich.edu

Department of Aerospace Engineering, University of Michigan,
Ann Arbor, Michigan, USA, 48109.

Abstract: Numerical shockwave anomalies for the Euler equations are directly related to the nonlinearity of the jump conditions and ambiguity of sub-cell shock position. In this work, flux functions are described that allow stationary shocks with no positional ambiguity. These functions are tested on several common model problems. In one dimension, they are shown to virtually eliminate both the carbuncle and slowly moving shockwave phenomena and reduce wall heating by as much as 60% with no loss of shock resolution.

Keywords: Numerical Algorithms, Computational Fluid Dynamics, Shock-Capturing, Flux Functions, Numerical Shockwave Anomalies

1 Introduction

When a physical shockwave is formed, it moves through the flow with a certain speed, having some finite width determined by physical dissipation until it encounters some event in its path. For numerical shockwaves, however, a numerical width is enforced, often much greater than the physical width. With this numerical width comes the formation of intermediate states having no direct physical interpretation. Even as the mesh is refined, these intermediate states do not go away; they simply occupy less space. The existence of intermediate states does raise some doubt, however, about how closely a captured shockwave may emulate an ideal discontinuous shockwave, or a real physical one.

There are in fact several types of error associated with intermediate shock states such as errors in shock position, spurious waves, or unstable shock behavior. These errors can be classified as *numerical shockwave anomalies*, numerical artifacts formed due to the presence of captured shockwaves within the flow solution. All of these numerical shockwave anomalies have been shown to be directly related to the nonlinearity of the jump conditions and to a resulting ambiguity in subcell shock position in the stationary shock [1]. By developing methods that do not suffer from this ambiguity, numerical shockwave anomalies can potentially be eliminated, or greatly reduced.

2 An Ambiguity in Shock Position

If the intermediate states are used to determine the subcell shock position using the equal area rule to compute shock position, the shock positions computed by density and energy are roughly equal for a weak shock but become increasingly divergent as Mach number increases. In fact, at high Mach numbers, the discrepancy is about $\Delta x/2$ for either the Godunov or Roe Fluxes [2]. The discrepancy tends to be smaller for more dissipative schemes and broader shocks, but it almost always remains. Our objective here is to remove the discrepancy without broadening the shock.

3 New Flux Functions

Given a set of conservation laws $\mathbf{u}_t + \mathbf{f}_x = \mathbf{u}_t + \mathbf{A}(\mathbf{u})\mathbf{u}_x = \mathbf{0}$, the first step in constructing our new flux functions is to define "interpolated fluxes" $\mathbf{f}_j^*$ that behave more smoothly near shocks than the actual fluxes. The interpolated fluxes are defined to have the following properties.

1. If the problem is linear so that the Jacobian matrix $\mathbf{A}(\mathbf{u})$ is constant, then $\mathbf{f}_j^* = \mathbf{f}_j$.
2. If the problem is nonlinear, but the data is smooth, then $\mathbf{f}_j^* = \mathbf{f}_j + \mathcal{O}(h^2)$
3. If the problem is nonlinear and involves a stationary shock, then $\mathbf{f}_j^*$ is constant, not only on each side of the shock, but also in the intermediate cell.

Then we can describe our new functions in the form

$$\mathbf{f}_{j+\frac{1}{2}} = \frac{1}{2}(\mathbf{f}_j^* + \mathbf{f}_{j+1}^*) - \frac{1}{2}\text{sign}(\tilde{\mathbf{A}}_{j+\frac{1}{2}})(\mathbf{f}_{j+1}^* - \mathbf{f}_j^*) \tag{1}$$

where $\tilde{\mathbf{A}}_{j+\frac{1}{2}}$ is the Roe matrix for $\mathbf{u}_j$ and $\mathbf{u}_{j+1}$. Note that the original Roe's Riemann Solver can be obtained if $\mathbf{f}_j^* = \mathbf{f}_j$. Hence, the new method recovers Roe's method for linear problems or smooth solutions. However, with $\mathbf{f}_j^*$ satisfying the above conditions, it can be shown that stationary shocks are captured, again with a single intermediate point, but now without any ambiguity in position.

We show results for such an $\mathbf{f}_j^*$. In Figure 1; for the Noh problem there is about a 60% reduction in the density error at the wall. No adverse effects are seen elsewhere. In Figure 2, we show contours of momentum in space time for a slowly moving shock. With the original solver, spikes in momentum are observed. The new flux removes the spikes, again without adverse effects.

4 Conclusions and Work to be Presented

In this work we develop new flux functions that utilize neighboring information to "work around" intermediate shock states and produce shockwaves with no subcell position ambiguities. These have shown promising results in one dimension in eliminating shockwave anomalies. A multi-dimensional extension and integration into a high-order framework is ongoing and will be presented.

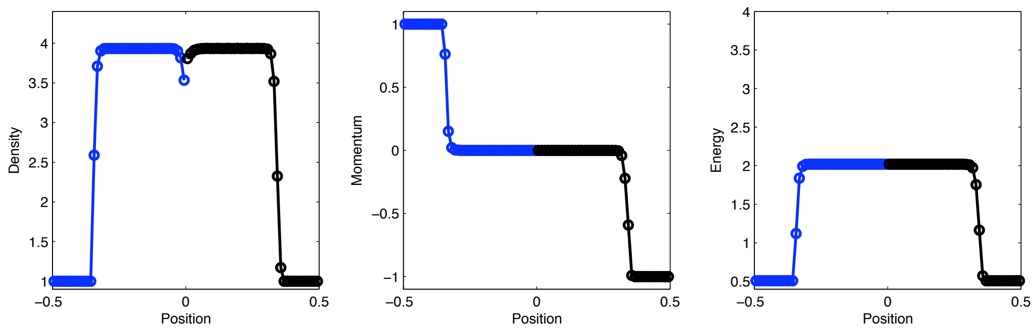

Figure 1: Conserved variables for the Noh problem. Results for Roe's Riemann Solver (blue) and our new flux function (black)

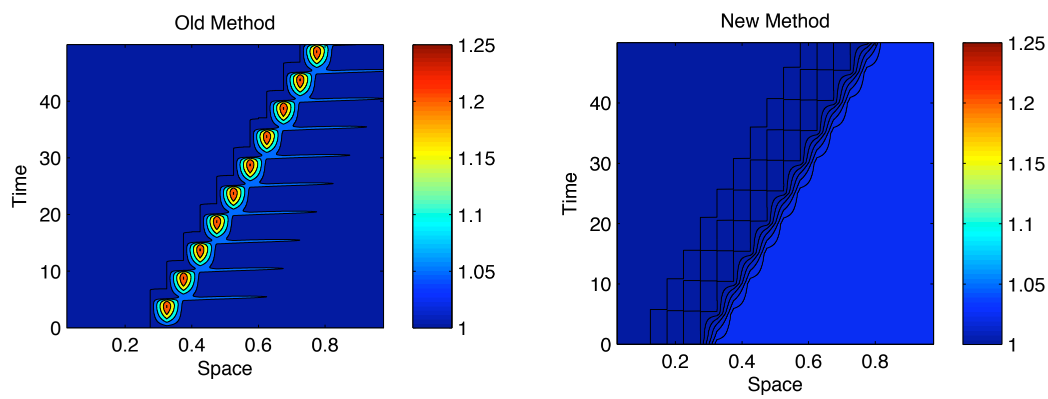

Figure 2: Contour plots of momentum for a slowly moving shockwave. Results for Roe's Riemann Solver (left) and our new flux function (right)

References

[1] Zaide, D. W. and Roe, P. L., "Shock Capturing Anomalies and the Jump Conditions in One Dimension," *Proceedings of 20th AIAA Computational Fluid Dynamics Conference, 2011-3686*, Jan 2011.

[2] Barth, T., "Some notes on shock resolving flux functions. Part 1: Stationary characteristics," 1989.