Adjoint-based Optimization of the Flapping Wing Performance

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Abstract: An adjoint-based time-dependent optimization methodology developed in [AIAA 2008-5857 and AIAA J. Vol.48, No.6, pp.1195-1206, 2010] is used to maximize the performance of a 3-D wing undergoing a flapping motion. The sensitivities of the thrust coefficient to wing kinematic and shape parameters are computed using the time-dependent discrete adjoint formulation. The unsteady discrete adjoint equations required for calculation of the sensitivity derivative are integrated backward in time. Our preliminary results on optimization of the 3-D unsteady flow near a pitching ONERA M6 wing show that the lift-to-drag ratio has been significantly increased after the optimization, thus indicating that this adjoint-based methodology is an efficient tool for optimization of essentially unsteady flows. Results on optimization of flapping wing flows will be presented in the final paper.

Keywords: Unsteady adjoint equations, Lagrange multipliers, time-dependent optimization, flapping wing, unsteady RANS equations.

1 Adjoint-based Time-Dependent Optimization Methodology

The turbulent flow near a flapping wing is modeled by the 3-D unsteady Reynolds-Averaged Navier-Stokes (RANS) equations with the Spalart-Allmaras turbulence model. The governing equations are solved using a second-order node-centered finite volume scheme on a tetrahedral body-fitted grid that rigidly moves along with the wing. To maximize the wing performance, the thrust coefficient is considered as a functional which is maximized by using optimal control theory, thus leading to the following discrete PDE-constrained optimization problem:

$$\begin{cases} \min F_{\text{obj}}(\boldsymbol{D}), \qquad F_{\text{obj}}(\boldsymbol{D}) = \sum_{n=1}^{N} f^n \,,\; f^n = \left(C^n - C^n_{\text{target}}\right)^2 \\ \text{subject to:} V^n \dfrac{3\boldsymbol{Q}^n - 4\boldsymbol{Q}^{n-1} + \boldsymbol{Q}^{n-2}}{2\Delta t} + \boldsymbol{R}^n + \boldsymbol{R}^n_{GCL}\boldsymbol{Q}^{n-1} = 0 \end{cases} \tag{1}$$

where $\boldsymbol{D}$ is a vector of the design variables, $\boldsymbol{Q}^{\boldsymbol{n}}$ is a vector of conservative variables, C^n *i*s an aerodynamic coefficient such as thrust, lift, or drag and C^n_{target} its target value, $\boldsymbol{R}^n$ is the spatial residual, and $\boldsymbol{R}^n_{GCL}$ is the geometric conservation law term. This discrete time-dependent optimization problem (1) is solved by the method of Lagrange multipliers which is used to enforce the governing equations as constraints. Differentiating the Lagrangian with respect to $\boldsymbol{D}$, the following equations for the flow adjoint variables $\boldsymbol{\Lambda}_f$ are derived:

$$\frac{3V^n\boldsymbol{\Lambda}^n_f - 4V^{n+1}{\boldsymbol{\Lambda}^{n+1}_f}^{n+1} + V^{n+2}\boldsymbol{\Lambda}^{n+2}_f}{2\Delta t} + \left[\frac{\partial \boldsymbol{R}^n}{\partial \boldsymbol{Q}^n}\right]^T \boldsymbol{\Lambda}^n_f + \boldsymbol{R}^n_{GCL}\boldsymbol{\Lambda}^{n+1}_f = -\left[\frac{\partial f^n}{\partial \boldsymbol{Q}^n}\right]^T \tag{2}$$

The grid adjoint equations are obtained in a similar way. The key advantage of the adjoint formulation is that the adjoint equations (2) are independent of $\boldsymbol{D}$, and should be solved once at each optimization iteration, regardless of the number of the design variables. Note that the unsteady adjoint equations (2) have to be integrated backward in time. With the adjoint variables satisfying the flow and grid adjoint equations, the sensitivity derivative is calculated as follows:

$$\frac{dL}{d\boldsymbol{D}} = \sum_{n=1}^{N} \left(\frac{\partial f^n}{\partial \boldsymbol{D}} + \left[\boldsymbol{\Lambda}_f^n\right]^T \left(\frac{\partial \boldsymbol{R}^n}{\partial \boldsymbol{D}} + \frac{\partial \boldsymbol{R}_{GCL}^n}{\partial \boldsymbol{D}} \boldsymbol{Q}^{n-1} \right) + \left[\boldsymbol{\Lambda}_g^n\right]^T \frac{\partial \boldsymbol{G}^n}{\partial \boldsymbol{D}} \right) \Delta t \quad (3)$$

A minimum of the objective functional is found by using a quasi-Newton gradient method. Further details of the implementation of this methodology can be found in [2].

2 Preliminary Results

The above adjoint-based methodology has been implemented in an unstructured 2nd-order, node-centered finite volume RANS code, FUN3D [2]. To assess the efficiency of the above methodology, optimization of the unsteady flow over the ONERA M6 wing undergoing pitching motion is considered as a test problem. The flow is assumed to be fully turbulent, and the eddy viscosity is modeled using the one-equation approach of Spalart and Allmaras. The freestream Mach and Reynolds numbers are set to be 0.3 and 10^6, respectively. The pitching reduced frequency nondimensionalized with respect to speed of sound is 0.009. The pitching takes place about a vector normal to the symmetry plane located 0.47 of the mean aerodynamic chord from the wing root leading edge. The pitching amplitude is 3^0. The design variables consist of a matrix of 55 camber parameters distributed across the wing. The objective functional is the lift-to-drag

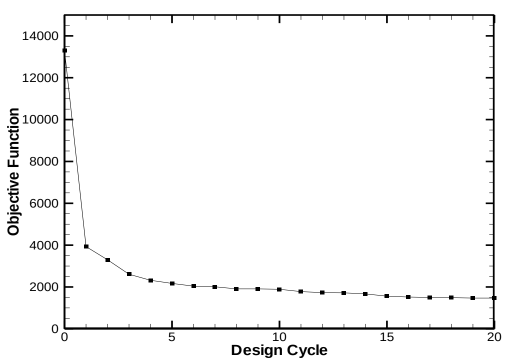

Figure 1. Objective functional history.

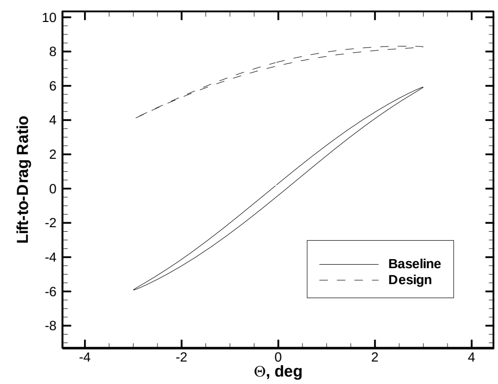

Figure 2. Lift-to-drag ratio before and after optimization

ratio, (*L/D*), defined over a physical time interval corresponding to the third pitch cycle. The target lift-to-drag ratio is set to 10 for each time step. A history of convergence of the objective functional is shown in Fig. 1. The objective functional has been reduced by 89% over 20 design cycles, with the majority of the reduction taking place during the first 5 cycles. The value of the lift-to-drag ratio obtained after optimization has been significantly increased as compared with that of the baseline wing, as one can see in Fig. 2, which demonstrates the efficiency of the present adjoint-based methodology.

3 Conclusion and Future Work

Results on optimization of flapping wing flows will be presented in the final paper.

References

[1] N. Yamaleev, B. Diskin, E. Nielsen, "Adjoint-based methodology for time-dependent optimization," AIAA Paper 2008-5857, 2008.

[2] E. Nielsen, B. Diskin, N. Yamaleev, "Discrete adjoint-based design optimization of unsteady turbulent flows on dynamic unstructured grids," AIAA J., Vol. 48, No.6, 2010.