Variational Multiscale Simulation of Flow along a Circular Cylinder with Exact Geometry

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Abstract: We present an application of variational multiscale (VMS) turbulence modeling methodology to the computation of laminar and turbulent flow along a circular cylinder. Isogeometric Analysis (IGA), based on Non-Uniform Rational B-Splines (NURBS) functions, is utilized in order to achieve higher-order approximation of the solution as well as exact geometry representation. We consider laminar and turbulent flows along a circular cylinder and demonstrate the applicability of the methodology to both regimes.

*Keywords:*Isogeometric Analysis, NURBS, Variational Multiscale, Turbulent Flow, Incompressible Navier-Stokes Equations.

1 Introduction

Isogeometric analysis is an analysis method that is utilizing the same basis function for geometry representation and solution approximation. Here, the Non-Uniform Rational B-spline (NURBS) functions are utilized for both solution approximation and model construction. By this way, we schieved exact geometry representation in addition to higher-order solution approximation. The incompressible Navier-Stokes equations are formulated and solved by the residual-based variational multiscale turbulence modeling[1].

2 Variational Multiscale Formulation

Variational multiscale formulation for solving Navier-Stokes euqations for incompressible flows can be represented as follows: Find $\mathbf{U}^h$ such that $\forall \mathbf{W}^h$

$$B^{MS}(\mathbf{W}^h, \mathbf{U}^h) = L^{MS}(\mathbf{W}^h) \quad (1)$$

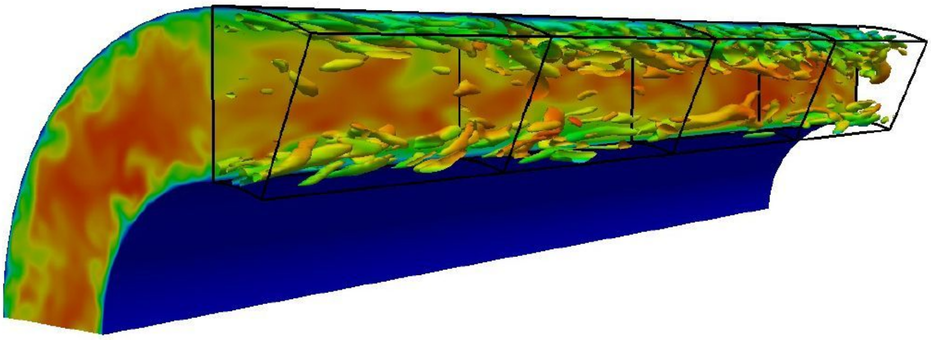

Figure 1: Turbulent flow at $Re_=8900$. Isosurface of $Q = 0.3$ colored by stream-wise velocity[2].

where

$$
\begin{aligned}
B^{MS}(\mathbf{W}^h, \mathbf{U}^h) &= B^G(\mathbf{W}^h, \mathbf{U}^h) \\
&+ \left(\mathbf{u}^h \cdot \nabla \mathbf{w}^h + \nabla q^h, \tau_M \mathbf{r}_M\right)_\Omega \\
&+ \left(\nabla \cdot \mathbf{w}^h, \tau_C r_C\right)_\Omega \\
&+ \left(\mathbf{u}^h \cdot (\nabla \mathbf{w}^h)^T, \tau_M \mathbf{r}_M\right)_\Omega \\
&- \left(\nabla \mathbf{w}^h, \tau_M \mathbf{r}_M \otimes \tau_M \mathbf{r}_M\right)_\Omega
\end{aligned}
\tag{2}
$$

$$
L^{MS}(\mathbf{W}^h) = (\mathbf{w}^h, \mathbf{f})_\Omega \tag{3}
$$

and

$$
\begin{aligned}
B^G(\mathbf{W}^h, \mathbf{U}^h) &= (\mathbf{w}^h, \frac{\partial \mathbf{u}^h}{\partial t})_\Omega + (\nabla^s \mathbf{w}^h, 2\nu \nabla^s \mathbf{u}^h)_\Omega - (\nabla \mathbf{w}^h, \mathbf{u}^h \otimes \mathbf{u}^h)_\Omega \\
&+ (q^h, \nabla \cdot \mathbf{u}^h)_\Omega - (\nabla \cdot \mathbf{w}^h, p^h)_\Omega
\end{aligned}
\tag{4}
$$

The superscripts MS and G stand for multiscale and Galerkin, respectively.

3 Preliminary Result

A snapshot of turbulent flow along a circular cylinder is presented in Figure 1.

References

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