Symmetric-conservative metric evaluations for higher-order finite difference scheme with the GCL identities on three-dimensional moving and deforming mesh

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Abstract: The use of standard evaluation for metric and Jacobian causes fatal errors in terms of freestream preservation on three-dimensional moving and deforming meshes. In this paper, new conservative forms are introduced for metric and Jacobian, which satisfy the geometric conservation law (:GCL) identity even when higher-order spatial discretization is employed for the moving and deforming meshes. The conservative quantities are ensured to keep constant for three-dimensional moving and deforming meshes with use of these new forms for the computation of the uniform flow. In addition, one of the new forms has spatial symmetricity, and some tests indicate the significance of the spatial symmetry in the expression of metric and Jacobian.

Keywords: geometric conservation law, volume conservation law, boundary-confirming coodinate transformations, body-fitted coordinates.

1 Introduction

The body-fitted coordinate system is often adopted when fluid motion around the arbitrary body shape is computed. In this case, the coordinate-transform metrics from body-fitted coordinate system into Cartesian coordinate system are introduced for the computation. Although transform metrics analytically satisfy the freestream preservation which is so-called "geometric conservation law" (GCL)[2], some discretized forms of metrics break the GCL identities with the use of finite-difference methods. The GCL identities consist of "surface closure law" (SCL) and "volume conservation law" (VCL)[5], and the VCL identity is focused in this paper. Some techniques for the discretization of time metrics and Jacobian were proposed in terms of geometries, e.g., finite-volume methods, which is not suitable for higher-order finite-difference methods. Meanwhile, the method to replace the governing equation by the one where Jacobian is split from the conservative-quantity fluxes[4]; however, values of time metrics and Jacobian do not satisfy the VCL identity for each moment. Such a governing equation is called *Split form* in convenience; the normal one is called *Non-split form.* It should be pointed out that the summation of conservative quantities among a closed region are not strictly conservative and vary from time to time when using *Split form.*

2 Problems and future works

We propose new analytical forms of time metrics and Jacobian: *asymmetric-* and *symmetric-conservative forms*, whose discretizations satisfy the VCL identity on every time step for any higher-order finite difference method. The Cartesian coordinate (t, x, y, z) and body-fitted coordinate (τ, ξ, η, ζ) are introduced under the assumption $t = \tau$; time metric $\partial\zeta/\partial t$ is expressed as ζ_t. Analytical expressions of ζ_t/J and J are

$$(\textit{Non-cons.}) \quad \zeta_t/J = x_\eta y_\tau z_\zeta - x_\eta y_\zeta z_\tau + x_\zeta y_\eta z_\tau - x_\zeta y_\tau z_\eta + x_\tau y_\zeta z_\eta - x_\tau y_\eta z_\zeta, \tag{1}$$

$$(\textit{Non-cons.}) \quad 1/J = x_\xi y_\eta z_\zeta - x_\eta y_\xi z_\zeta + x_\zeta y_\xi z_\eta - x_\xi y_\zeta z_\eta + x_\eta y_\zeta z_\xi - x_\zeta y_\eta z_\xi, \tag{2}$$

or

$$(\textit{Asym-cons.}) \quad \zeta_t/J = [\{(x_\eta y)_\xi - (x_\xi y)_\eta\}z]_\tau + [\{(x_\tau y)_\eta - (x_\eta y)_\tau\}z]_\xi + [\{(x_\xi y)_\tau - (x_\tau y)_\xi\}z]_\eta, \tag{3}$$

$$(\textit{Asym-cons.}) \quad 1/J = [\{(x_\xi y)_\eta - (x_\eta y)_\xi\}z]_\zeta + [\{(x_\zeta y)_\xi - (x_\xi y)_\zeta\}z]_\eta + [\{(x_\eta y)_\zeta - (x_\zeta y)_\eta\}z]_\xi, \tag{4}$$

where J is Jacobian determinant of coodinate transformation; *Non-cons.* and *Asym-cons.* denote *non-* and *asymmetric-conservative form*, respectively. The *symmetric-conservative forms* can be introduced in terms of coordinate invariance on basis of *Asym-cons.* which are omitted for brevity. Some tests are carried out on three-dimensional moving and deforming meshes with 6th-order compact-differencing scheme to confirm the validation of these new forms for time metrics and Jacobian; first, the uniform flow is calculated for the preservation error of conservative quantities; second, two-dimensional isentropic vortex is calculated to examine the resolution of new schemes and clarify the trend due to symmetry property in the discretization of time metrics and Jacobian.

Geometrical interpretations for *non-*, *asymmetric-* and *symmetric-conservative forms* from the viewpoint of finite-volume methods are now under construction. For the use of moving and deforming mesh, adequate forms for spatial metrics in time-space combined system[1] are also pursued on the basis of *symmetric-conservative forms.*

References

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