High Order Residual Distribution Scheme for the RANS Equations

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Abstract: In this work the use of Residual Distribution schemes for the discretization of conservation laws is described. The scheme uses a Lagrangian interpolation of the solution over the element to achieve high order spatial accuracy for steady problems. The formulation is compact and easily generalizable to all spatial dimensions and to all type of elements.
The accuracy of the numerical scheme is proved by numerical experiments on benchmark problems in two and three spatial dimensions. Furthermore, the results for the Reynolds Averaged Navier-Stokes computation, based on the Spalart-Allmaras model, of compressible turbulent flows are reported.

Keywords: High Order Schemes, Residual Distributions, RANS Equations, Compressible flows.

1 Introduction

Residual Distribution schemes represent a very interesting alternative to Discontinuous Galerkin (DG) schemes [3]. While computationally compact and probably more flexible, DG schemes suffer from the serious drawback of a very fast growth of the number of degrees of freedom with the cell polynomial degree. In RD schemes the formulation remains local, as in DG, but the number of degree of freedom grows less quickly. The price to pay is to impose a continuous approximation of the solution, even though some papers report their extension to discontinuous approximation. [4, 5]

Results for RD schemes in the case of order more that two have been presented for the system of the Euler equations in [1, 2], but the high order discretization of the Navier-Stokes equation is still an open question. In the RD framework the viscous terms have been discretized using a Galerkin finite-element formulation, unfortunately this choice does not guarantee a uniform order of accuracy on all the range of the local Reynolds numbers [7]. By resorting the Petrov-Galerkin analogy [6] is possible to construct a RD scheme for the discretization of the viscous terms with an uniform order of accuracy, but this is limited to the case of a third order scheme on triangular mesh.

In this work the discretization of the viscous problems is obtained by first computing the total residual of both the advective and diffusive terms and then distributing it to all the degrees of freedom of the element within the same RD formulation. The resulting scheme is high order accurate and can be formulated on hybrid grids in two or three spatial dimensions. Furthermore due to the compactness of the formulation the numerical scheme has an excellent parallel scalability, which makes it well suited for large-scales computations on parallel computers.

To asses the validity of the approach several numerical experiments are performed. Underlining simulations of inviscid flows are carried out to verify the accuracy of the discretization of the advective terms. Within this part, improvements to the existing scheme are introduced. The matrix-free technique is used to accelerate the convergence of the implicit scheme to the steady state solutions. The scheme is extended to the high order representation of the boundaries to minimize the solution errors produced by a linear representations of the curved boundaries. Numerical simulations of laminar viscous flows are performed and is proposed an improved methodology for the imposition of the boundary conditions on adiabatic walls.

The implementation of the Spalart-Allmaras RANS model is discussed, including some modifications of the original formulation to enhance the stability of the numerical scheme. Standard test cases are performed to verify the correctness of implementation.

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