A Numerical Study of Chaotic Dynamics in Thermal Ignition and Reactive Swirling Flow

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Abstract: Nonlinear dynamical phenonomena in combustion processes is an active area of experimental and theoretical research. This is in large part due to increasingly strict environmental pressures to make gas turbine engines and industrial burners more efficient. Using numerical methods, this study examines chaotic dynamics in a thermal ignition framework as well as axisymmetric swirling flow leading to enhanced mixing with vortex breakdown. The incompressible and compressible, reactive Navier-Stokes equations in terms of stream function, vorticity, circulation are used together with a Poisson's equation for the pressure. Results, details of the numerical algorithms, as well as numerical verification techniques and validation with sources from the literature will be presented.

Keywords: Combustion, Ignition, Vortex Breakdown, Chaos.

1 Introduction

Combustion is an interdisciplinary field combining elements from fluid dynamics, chemical kinetics and transport phenomena. The general problem is governed by the reactive, time dependent, compressible Navier−Stokes equations for N chemical species. Difficulties that will be addressed are multiple time scales, non-linearity (both in the advection and chemical source terms) and combustion instabilities in transitions to unsteady oscillatory and aperiodic solutions.

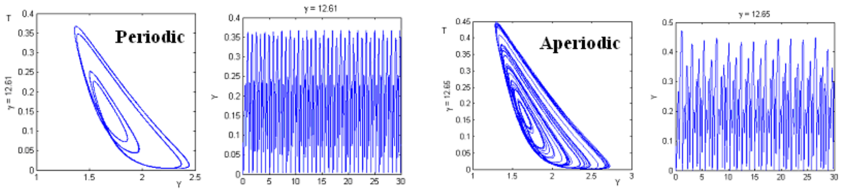
2 Problem Statement

2.1 Thermal Ignition Theory

The choice to study thermal ignition theory is proposed because of issues related to bifurcations and chaotic dynamics and to improve numerical capabilities to study stiff chemical reaction problems in fluid dynamics. If we assume a single chemical species, Y, $\vec{V} = 0$ (mainflow) and constant density then this gives the nonlinear advection-reaction-diffusion model:

$$\frac{\partial Y}{\partial t} = D\nabla^2 Y - \delta Y e^{-\theta/T}, \qquad \frac{\partial T}{\partial t} = \kappa\nabla^2 T + \delta A Y e^{-\theta/T}$$

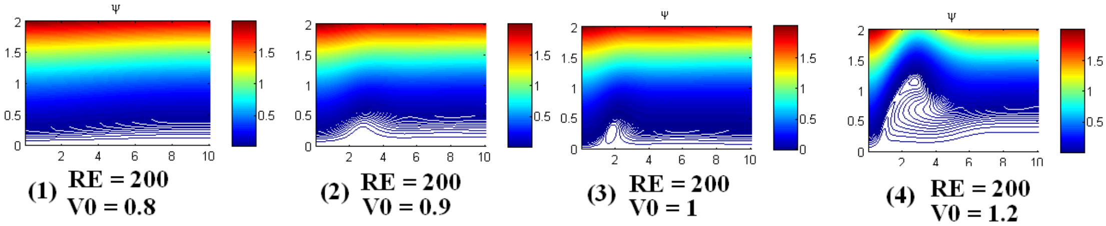
with constant parameters: $\kappa, D, \delta, A, \theta$. We consider this problem in 1 and 2 spatial dimensions. The Lewis number (Le) is the non-dimensional ratio of mass to thermal diffusivity (k/D). This problem is known to have periodic and aperiodic solutions, with examples computed here:



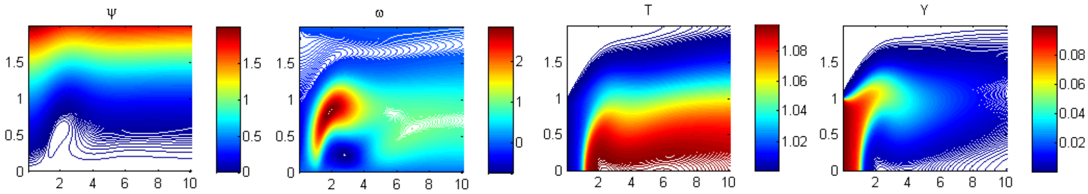
Typically a robust implicit method is needed to study this problem. The solution strategies we considered are IMEX operator splitting methods MCNAB and 2-SBDF [7]. Having two strategies that produce the same result are used along with convergence tests as verification tools to check formal accuracy. To study oscillations and chaotic dynamics, tools from time series and nonlinear analysis will be used, such as for example, Lyapunov exponent and Melnikov function. Selected results of special cases are available to validate the codes [3, 6].

2.2 Axisymmetric Swirling Flow

Swirling the flow is a control technique whose main effects include considerably enhancing stability of most flames by creating toroidal recirculation zones facilitating mixing, decreasing combustion length through fluid entrainment, and even reducing maintenance costs on equipment [1]. A strongly swirling flow, sufficient to induce recirculation by vortex breakdown is needed to achieve these benefits. Vortex breakdown as a fluid mechanics problem has received extensive research attention [5]. Benchmarks for the steady cases and known chaotic dynamics in closely related problems will be studied for validation [4, 8]. The problem is governed by the compressible, reactive equations Navier-Stokes equations. A code has been developed and validated for the non-reactive case following [8]. The problem considered is axisymmetric flow in a cylindrical geometry with a steady, continuous, vortex generated at the inlet. Comprehensive results, in the absence of combustion, have been reproduced. Here is an example showing vortex breakdown for $Re = 200$, and a range of the swirl parameter, $V_0 = 0.8, 0.9, 1.0, 1.2$ respectively:



The model has been extended to include combustion in the compressible reactive case. A preliminary test case for small heat release with constant pressure has been calculated:



Vortex breakdown develops sooner with the presence of combustion and this matches known results [2]. It is observed that the mixing due to vortex breakdown greatly affects the temperature such that a local temperature maximum is coincident with a vortex bubble center. Understanding how instabilities are effected by geometry and reactant consumption are the main goals of this study and several examples will be presented in the final paper.

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