

# Numerical Study on Leading Edge Receptivity of the Flat Plate Boundary Layer to Vortical Disturbance

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**Abstract:** The leading edge receptivity to vorticity disturbance is investigated by a three-dimensional numerical simulation. The disturbances are given by a two dimensional periodic vorticity fluctuation at the upstream boundary as a boundary condition. The vorticity fluctuation inside the boundary layer becomes more intense when the vertical scale of the oncoming vorticity fluctuation is larger. It is shown that the tangential velocity induced at the stagnation point has a strong influence on the receptivity.

*Keywords:* Receptivity, Elliptic Leading Edge, Vorticity

## 1 Introduction

The concept of the receptivity was proposed by Morkovin[1] as a key problem in a boundary layer transition process. Because the boundary layer is a vorticity layer, this study focuses on the changes in the vorticity fluctuation pattern to understand the receptivity. In this study, the leading edge receptivity against incoming vortical disturbances is numerically investigated, focusing on the deformation of vorticity patterns inside a boundary layer.

## 2 Numerical method

Three-dimensional incompressible Navier-Stokes equations are solved by the finite difference method using a body-fitted coordinate on a regular grid system. A third-order upwind difference scheme is used in the convection terms. For the other terms, the second-order central difference scheme is employed. The third-order Adams-Bashforth explicit scheme is used for the convection terms and the Crank-Nicolson implicit scheme is applied to the viscous terms. In addition, the multi-directional finite difference scheme is used for the discretization of the all terms in the N-S equations. Figure 1 shows the computational domain around a flat plate with an elliptic leading edge of an aspect ratio of 1:5, where  $a$  is the leading edge length and  $b$  is the half of the thickness of the flat plate. The origin of the Cartesian coordinate system is set at the tip of the leading edge of the plate, where  $x$ ,  $y$  and  $z$  axes denote the streamwise, vertical and spanwise directions, respectively. The numbers of grids are 449 points in  $\xi$  direction, 193 points in  $\eta$  direction, and 6 points in  $\zeta$  direction. The flat plate length is four times larger than the length of the leading edge. Reynolds number based on the leading edge length  $a$  and the freestream velocity  $U_\infty$  is  $4.0 \times 10^4$ . The spanwise length of the calculation region is  $b$ .

As for the velocity boundary conditions, the non-slip condition is imposed at the wall, the Sommerfeld radiation condition is applied at the outlet boundary and the Dirichlet condition is enforced at the upper and lower boundaries. As for the pressure boundary conditions, the Dirichlet condition is used at all boundaries and at the wall. The pressure averaged over the calculation field is adjusted to be unity. After the base flow becomes steady, two-dimensional disturbances are added to the freestream by periodically changing only the streamwise velocity  $u$  at the upstream boundary. The following equation describes the disturbance,

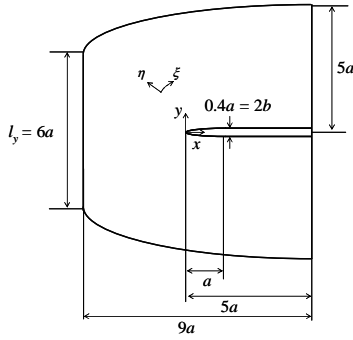


Figure 1 Computational domain

$$\frac{u'}{U_\infty} = \begin{cases} A \times \sin(2\pi ft) \times (1 + 0.5/\alpha) \frac{s}{y} \left( 1 - \exp\left(-\alpha \left(\frac{y}{s}\right)^2\right) \right) & -\frac{l_y}{2} \leq \frac{y}{a} \leq \frac{l_y}{2} \\ 0 & \text{otherwise} \end{cases}$$

where  $f$  is the non-dimensional frequency,  $l_y$  is the length of the oscillation region,  $A$  is the amplitude,  $\alpha = 1.25643$ [2], and  $s$  is the parameter corresponding to the vortex core radius. Also,  $t$  is the time. In this study, the simulations are performed for three different scales of  $s/b$ , which are 1, 2 and 3. The amplitude of the introduced fluctuation  $A$  is 1% of the freestream velocity and the non-dimensional frequency  $f$  is 1.

### 3 Numerical Results and Discussion

The RMS values of the velocity fluctuations in the wall-tangential direction measured at the grid points next to the wall are plotted in Fig.2. Figure 2 shows that the velocity fluctuations in the wall-tangential direction become larger in proportion to the scale parameter  $s$  downstream. There is also a noticeable difference between the  $u'_{\text{rms}}$  at the stagnation point depending on  $s/b$ . It should be noted that  $u'_{\text{rms}}$  at the stagnation point corresponds to the  $v$  component velocity there. Figure 3 depicts the RMS values of the vertical velocity fluctuations  $v'_{\text{rms}}$  in the freestream along the  $y/a = 0$  line. It can be found that the  $v'_{\text{rms}}$  reaches its peak very close to the leading edge just before it is damped by viscosity. It is also shown that  $v'_{\text{rms}}$  is influenced by the vortex core radius  $s$ . These results imply that the leading edge receptivity is governed by the periodic tangential velocity fluctuations at the stagnation point.

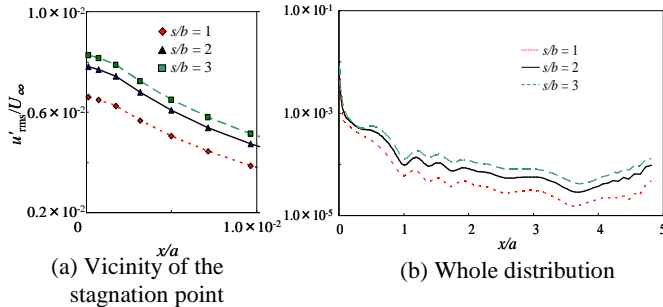


Figure 2 Distribution of the tangential velocity fluctuations near the wall

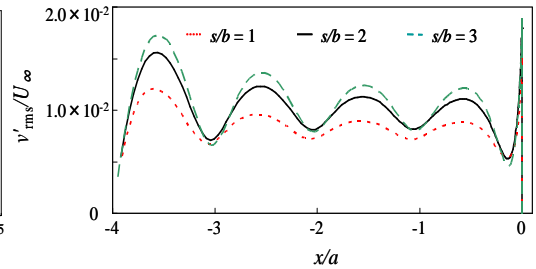


Figure 3 Distribution of the vertical velocity fluctuations in the freestream along  $y = 0$

### 4 Conclusion

A numerical study is performed to investigate a relation between the vorticity disturbances and the leading edge receptivity. Freestream disturbances are given as a two dimensional periodic vorticity fluctuation at the upstream boundary. The result shows that the velocity fluctuation inside a boundary layer becomes more intense when the vertical scale of the oncoming vorticity fluctuation is large. It is found that the tangential velocity fluctuations near the wall at the stagnation point strongly affect the amplitude of velocity fluctuations inside a downstream boundary layer.

### References

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