Monolithic Solver in ALE Framework for Interaction of Incompressible Fluid and Elastic Structure

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Abstract: In the paper a monolithic scheme for solution of a FSI problem is introduced. ALE formulation is used to account for the deformation of the computational domain and FEM to discretize the fluid and the structural equations in space. BDDC method is applied as a preconditioner in a Newton-Krylov type scheme for solution of the nonlinear system.

Keywords: Fluid-Structure Interaction, ALE Method, BDDC.

1 Introduction

ALE method remains one of the most effective approaches to solution of the fluid-structure interaction problems provided the deformations are not extremely large (180° rotation etc.). Partitioned schemes in ALE framework are probably method of choice where compressible fluid is considered, for incompressible fluid the added mass effect is known to rule out weak coupling and require very small time-step increasing computational costs in case of implicitly coupled schemes. Therefore, monolithic solutions come to focus.

2 Problem Statement

A FSI problem for an incompressible viscous fluid modeled by Navier-Stokes equations in ALE form and linearly elastic structure is to find fluid velocity $\mathbf{v}^F$, fluid pressure p, structural displacement $\mathbf{d}^S$ and mesh displacement $\mathbf{d}^G$ satisfying system

$$\begin{aligned}
\frac{\partial \mathbf{v}^F}{\partial t} + [(\mathbf{v}^F - \mathbf{v}^G)\cdot\nabla]\mathbf{v}^F - \nu\Delta\mathbf{v}^F + \nabla p &= \mathbf{f}^F, \\
\mathrm{div}\,(\mathbf{v}^F - \mathbf{v}^G) &= 0 \quad \text{in } \Omega^f, \\
\rho^S \frac{\partial^2 \mathbf{d}^S}{\partial t^2} + \nabla\cdot\sigma(\mathbf{d}^S) &= \mathbf{f}^S \quad \text{in } \Omega^s, \\
\Delta \mathbf{d}^G &= 0 \ \text{ in } \ \Omega^f, \\
\frac{\partial \mathbf{d}^S}{\partial t} &= \mathbf{v}^F \text{ on } \Gamma_I, \\
\tau\cdot\mathbf{n}^F &= \sigma\cdot\mathbf{n}^S \text{ on } \Gamma_I, \\
\mathbf{d}^G &= \mathbf{d}^S|_{\Gamma_I} \ \text{ on } \ \Gamma_I,
\end{aligned} \tag{1}$$

where Ω^f, Ω^s are fluid and structural domains respectively and Γ_I is an interface through which the fluid and the structure interact. Assuming fully implicit time discretization for both the fluid and the structure, a nonlinear strong coupling problem arises which is solved in a monolithic way.

3 Newton-Krylov scheme with BDDC preconditioning

To solve this nonlinear FSI problem fluid, structural and mesh equations are discretized in space by FEM to obtain a system for nodal values in residual form

$$\mathbf{F}^{FSI}(\mathbf{v}^F, p, \mathbf{d}^S, \mathbf{d}^G) = \mathbf{0}, \tag{2}$$

which is linearized by the Newton method into a system

$$\mathbf{J}^{FSI}\Delta\mathbf{x} = -\mathbf{F}^{FSI}, \tag{3}$$

where $\mathbf{J}^{FSI}$ is the Jacobian matrix. To effectively solve the linearized problem using Krylov type method a preconditioner $\mathbf{M}$ is applied to (3) to state

$$\mathbf{J}^{FSI}\mathbf{M}^{-1}\mathbf{M}\ \Delta\mathbf{x} = -\mathbf{F}^{FSI}. \tag{4}$$

As a preconditioner BDDC method [1, 2] is used with the aim of parallelization of the solution procedure.

References

[1] J. Mandel, and B. Sousedík. BDDC and FETI-DP under minimalist assumptions. *Computing*, 81:269-280, 2007.

[2] J. Šístek, B. Sousedík, P. Burda, J. Mandel, and J. Novotný. Application of the parallel BDDC preconditioner to the Stokes flow. *Comput. Fluids*, 46:429-435, 2011.