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Variational Multiscale Simulation of Flow around a Circular Cylinder with Exact Geometry

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Abstract: We present an application of variational multiscale (VMS) turbulence modeling methodology to the computation of laminar and turbulent flow around a circular cylinder. Isogeometric Analysis (IGA), based on Non-Uniform Rational B-Splines (NURBS) functions, is utilized in order to achieve higher-order approximation of the solution as well as exact geometry representation. We consider laminar and turbulent flows around a circular cylinder and demonstrate the applicability of the methodology to both regimes.

Keywords: Numerical Algorithms, Computational Fluid Dynamics, Turbulence Modeling, Aeroacoustics.

1 Introduction

Isogeometric analysis is an analysis method that is utilizing the same basis function for geometry representation and solution approximation. Here, the Non-Uniform Rational B-spline (NURBS) functions are utilized for both solution approximation and model construction. By this way, we achieved exact geometry representation in addition to higher-order solution approximation. The incompressible Navier-Stokes equations are formulated and solved by the residual-based variational multiscale turbulence modeling[1].

2 Variational Multiscale Formulation

Variational multiscale formulation for solving Navier-Stokes euqations for incompressible flows can be represented as follows: Find $\mathbf{U}^h$ such that $\forall \mathbf{W}^h$

$$B^{MS}(\mathbf{W}^h, \mathbf{U}^h) = L^{MS}(\mathbf{W}^h) \tag{1}$$

where

$$\begin{aligned} B^{MS}(\mathbf{W}^h, \mathbf{U}^h) \;\; = \;\; & B^G(\mathbf{W}^h, \mathbf{U}^h) \\ + \;\; & \left(\mathbf{u}^h \cdot \nabla \mathbf{w}^h + \nabla q^h, \tau_M \mathbf{r}_M\right)_\Omega \\ + \;\; & \left(\nabla \cdot \mathbf{w}^h, \tau_C r_C\right)_\Omega \\ + \;\; & \left(\mathbf{u}^h \cdot (\nabla \mathbf{w}^h)^T, \tau_M \mathbf{r}_M\right)_\Omega \\ - \;\; & \left(\nabla \mathbf{w}^h, \tau_M \mathbf{r}_M \otimes \tau_M \mathbf{r}_M\right)_\Omega \end{aligned} \tag{2}$$

$$L^{MS}(\mathbf{W}^h) = (\mathbf{w}^h, \mathbf{f})_\Omega \tag{3}$$

and

$$\begin{aligned} B^G(\mathbf{W}^h, \mathbf{U}^h) &= (\mathbf{w}^h, \frac{\partial \mathbf{u}^h}{\partial t})_\Omega + (\nabla^s \mathbf{w}^h, 2\nu \nabla^s \mathbf{u}^h)_\Omega - (\nabla \mathbf{w}^h, \mathbf{u}^h \otimes \mathbf{u}^h)_\Omega \\ &+ (q^h, \nabla \cdot \mathbf{u}^h)_\Omega - (\nabla \cdot \mathbf{w}^h, p^h)_\Omega \end{aligned} \tag{4}$$

The superscripts MS and G stand for multiscale and Galerkin, respectively.

3 Isogeometric Analysis Concepts

Isogeometric analysis is a computational mechanics technology that uses the same basis for analysis as is used to describe the geometry [2]. The basic idea is to use in analysis the basis-function technology used in computational geometric representations. In modern Computer Aided Design (CAD) systems, Non-Uniform Rational B-splines (NURBS) are the dominant technology. When a NURBS model is constructed, the basis functions used to define the geometry can be systematically enriched by h-(knot insertion), p-(order elevation), and/or k- refinement, without changing the geometry or its parameterization. Our first goal by using IGA is the construction of the exact model. Toward to this goal, NURBS quadratics and cubic basis functions are employed. The second important achievement by using IGA will be, preserving C^1-continous and C^2-continous across element interfaces regarding to employ quadratic and cubic NURBS elements respectively. The approach ensures C^1-continuity in the solution (velocity, pressure, etc.) across element interfaces inside every patches while C^0-continuity across the patch boundary. The C^1-continuity in the solution across the patch boundary has been implemented by applying a constraint equation to set periodic basis functions. Figure 1-a, shows the two quadratic NURBS patches which meet on an interface and their bases generated using open knot vectors. If two knot vectors formed open knot vectors are brought together, they can be made to act as one if the control variables (velocity and pressure) of the two functions on their interface are always equal to each other. The result is the same as the case of a single knot vector with C^0 boundary at the interface, Figure 1-b. To preserve C^1-continuous boundary at the interface, periodic basis functions can be used shown in Figure 1-c. In this way C^1 is preserved everywhere even on patch boundaries.

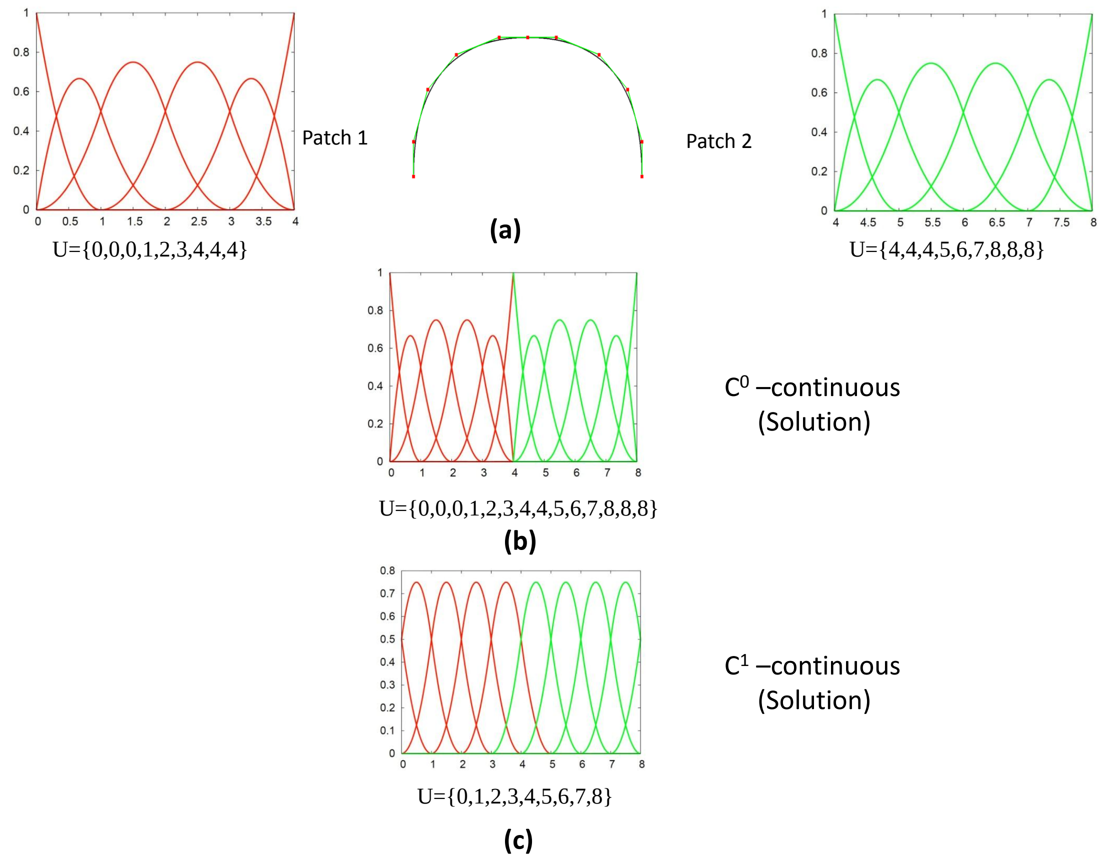

Figure 1: Improving continuity with quadratic NURBS between adjacent patches. (a) If two knot vectors formed open knot vectors are brought together, they can be made to act as one if the control variables (velocity and pressure) of the two functions on their interface are always equal to each other. Red points and green lines represent the control points and control edges, respectively. (b) The result is the same as the case of a single knot vector with C^0 boundary at the interface. (c) By using periodic basis functions we can keep C^1-Continuous is preserved every where even in patch boundaries.

4 Flow along Circular Cylinder

We conduct numerical experiments for the laminar and turbulent concentric annulus flow at bulk-flow Reynolds number of 0.004 and 8900 for the full domain, respectively. Figure 2 shows turbulent flow at $Re = 8900$, based on bulk-flow Reynolds number. The flow is driven by a pressure gradient, f_x, acting in the stream-wise direction. The value of kinematic viscosity ν is set to 4.49438×10^{-4} and, in order to maintain a constant flow rate, the forcing f_x is adjusted. Quadratic NURBS are utilized in all the computations [2]. We perform our simulations using a sequence of h-refined meshes to assess the convergence properties of the numerical methodology. The continuity of the basis functions is kept at C^1, which is the maximal continuity achievable for a quadratic NURBS discretization. We note that at each level of refinement, quadratic NURBS capture the problem geometry exactly. The coarsest mesh computations were performed on a mesh of $64 \times 16 \times 16$ elements in the circumferential, radial and axial directions, respectively. With each h-refinement step we double the number of elements in each parametric direction to achieve our finest discretization of $256 \times 64 \times 64$ elements. A uniform mesh is used in the circumferential and axial directions. In the radial direction, the meshes are obtained by distributing the knots according to a hyperbolic tangent function to better capture the boundary layer turbulence. The velocity profile in wall units is presented in Figure 2. A snapshot of turbulent flow along a circular cylinder is presented in Figure 3.

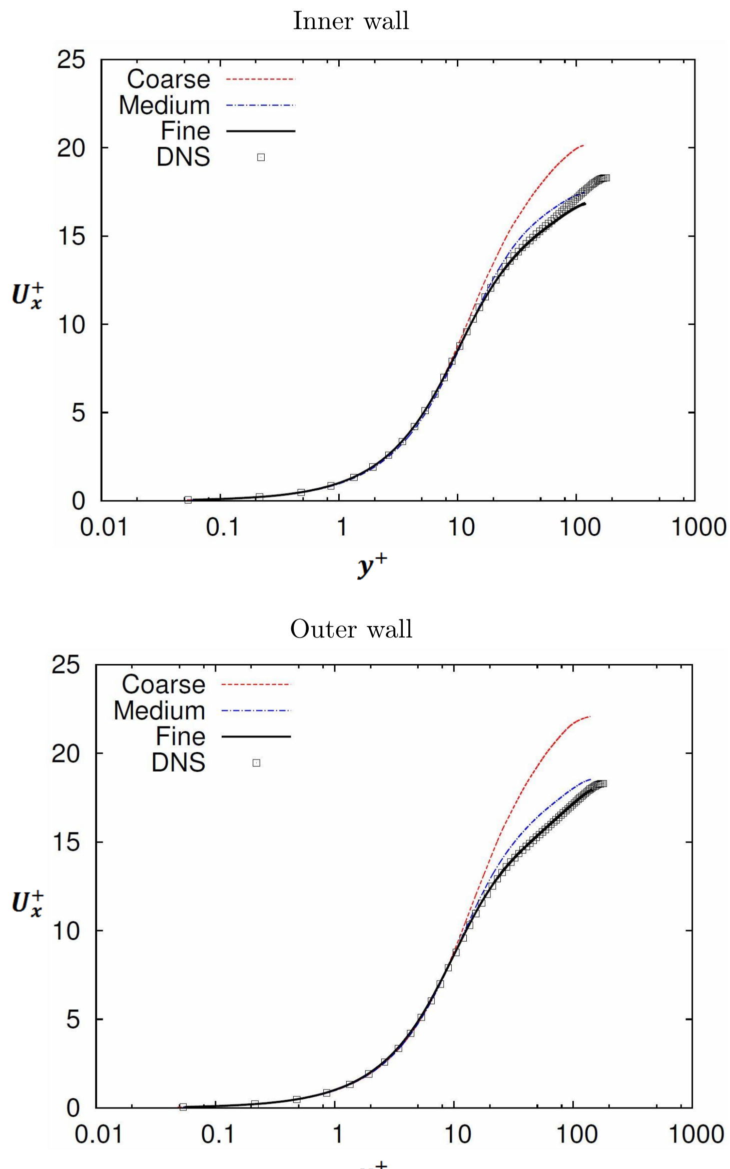

Figure 2: Inner and outer wall mean velocity distributions at $Re = 8900$ computed using quadratic NURBS: h-refinement interpretation of results. Here $U_x^+ = \frac{U_x}{u_\tau}$ and $y^+ = \frac{y u_\tau}{\nu}$. DNS: Kim *et al.* [3].

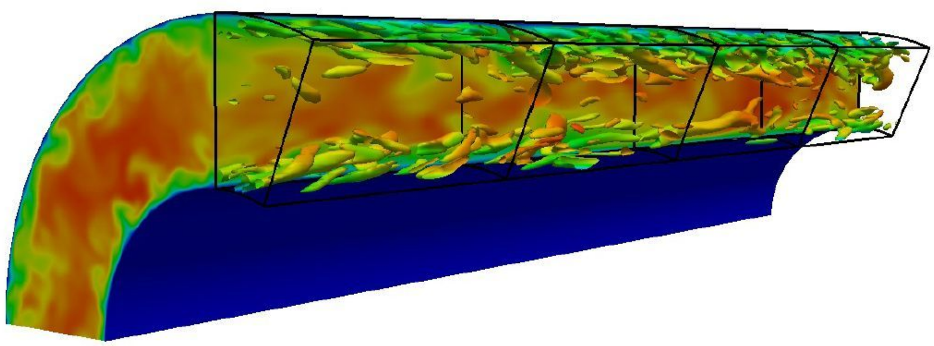

Figure 3: Turbulent flow at $Re = 8900$. Isosurface of $Q = 0.3$ colored by stream-wise velocity[4].

5 Flow around a Circular Cylinder

We consider the computational domain with multiple NURBS patches therefore the domain is divided into 8 patches. The domain shown in Figure 4 has an inner radius of $R_i = 0.5$, and an outer radius of $R_o = 20$. The length of the cylinder is $L = 10$. Also the geometric models have been constructed based on quadratic NURBS basis functions in the all directions. Non-dimensional Navier-Stokes equations were solved. Uniformed inflow was assumed at the inlet boundary, while traction free boundary was employed at outlet which allowed effective flow passage without any reflection as shown in Figure 4. Simulation were carried out at Reynolds number 10, 50, 100, 150 and 1000 to analyze different patterns of reflective of laminar flow regime. In Figure 5 green points and black lines represent control points (or control values for solutions e.g. pressure and velocity) and control edges, respectively and blue solid shows a quadratic element. Figure 6 shows the comparison of drag coefficient between our numerical results and experimental results [5].

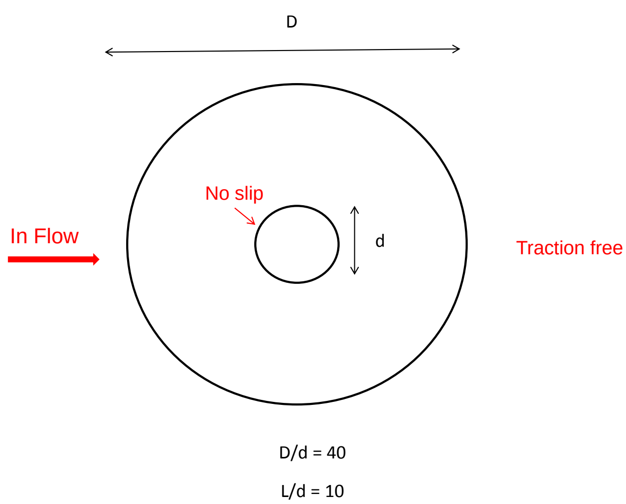

Figure 4: Uniformed inflow was assumed at the inlet boundary, while traction free boundary was employed at outlet which allowed effective flow passage without any reflection.

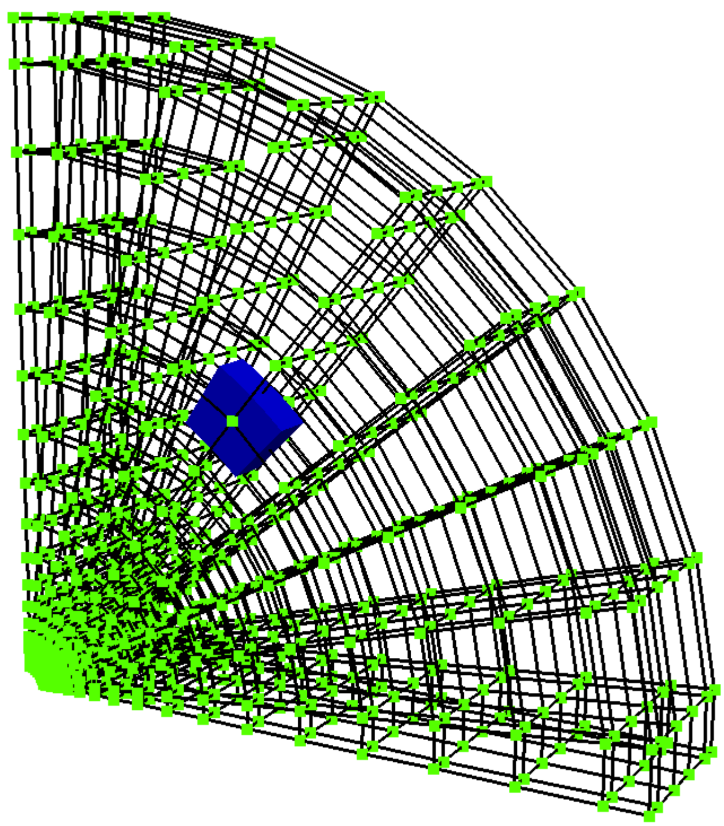

Figure 5: Green points and black lines represent control points (or control values for solutions e.g. pressure and veloticy) and control edges, respectively. Number of control points $10 \times 18 \times 4$ in the circumferential, radial and axial directions, respectively for each patch. Blue solid shows a quadratic NURBS element.

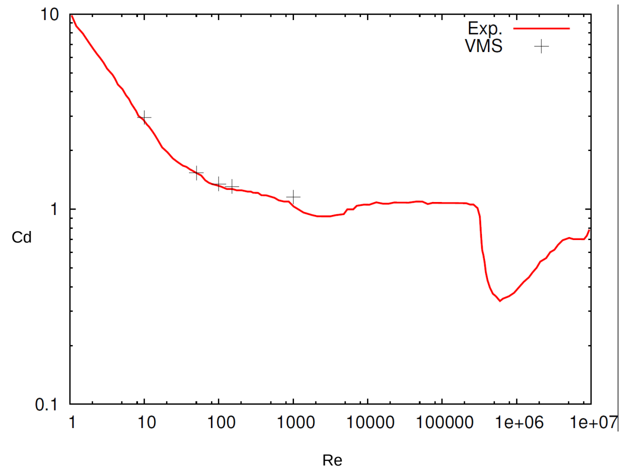

Figure 6: Comparison of Drag coefficient between present study and experiment data [5].

6 Conclusion

Flow around a circular cylinder is simulated by using the variational multiscale (VMS) formulation and isogeometric Analysis (IGA), based on Non-Uniform Rational B-Splines (NURBS) functions. Through the unified VMS formulation, both laminar and turbulent flows are simulated without extra modification for accouting turbulence effect. Quadratic NURBS element is utilized in order to achieve higher-order approximation of the solution as well as exact geometry representation. Laminar and turbulent flows both along and across a circular cylinder are simulated and the validity of the approach(VMS/IGA) is demonstrated.

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