

# Airfoils Admitting Anomalous Behavior of Lift Coefficient in Descending Transonic Flight

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**Abstract:** Airfoils admitting abrupt changes of the lift coefficient at small variations of free-stream parameters are considered. A numerical simulation of transonic flow is based on the RANS equations using BSL Reynolds stress and  $k - \omega$  SST turbulence models. The study demonstrates the existence of adverse free-stream conditions for the descending flight of Boeing 737 Outboard, J-78, and Whitcomb airfoils.

*Keywords:* Turbulent Flow, Local Supersonic Regions, Interaction, Instability.

## 1 Introduction

In the last two decades, a number of numerical studies showed instability of local supersonic regions on airfoils that are flat or nearly flat in the midchord region. Such airfoils admit jumps of the lift coefficient  $C_L$  due to abrupt changes of the pressure distribution on the airfoil at slight variations of the angle of attack or free-stream Mach number [1, 2, 3]. In case of symmetric airfoils, in addition to the jumps, there are self-exciting oscillations of  $C_L$  due to the instability of shock-induced boundary-layer separation [3].

In this paper, we focus on asymmetric airfoils at small positive or negative angles of attack, which are typical for a descending flight of civil/transport aircraft. Also, small angles of attack occur in cruise flight due to gusts or atmospheric turbulence. We show that the airfoil response to small perturbations can be anomalously strong.

## 2 Problem Formulation and a Numerical Method

We consider a lens-shaped computational domain bounded by two circular arcs,  $\Gamma_1$  and  $\Gamma_2$ , and by an airfoil placed at the center of the domain (see Fig. 1). The width and height of the domain are 80 and 200 chord lengths, respectively. On the inflow part  $\Gamma_1$  of the boundary, we prescribe stationary values of the angle of attack  $\alpha$ , the Mach number  $M_\infty < 1$ , and the static temperature  $T_\infty$ . On the outflow part  $\Gamma_2$  of the boundary, the static pressure  $p_\infty$  is given. The no-slip condition and vanishing flux of heat are used on the airfoil. The specific heat at constant pressure  $c_p$  is 1004.4 J/(kg·K), while the one at constant volume  $c_v$  is 717.3 J/(kg·K). The molar mass and molecular dynamic viscosity are 28.96 kg/kmol and  $1.831 \times 10^{-5}$  kg/(m·s), respectively. Initial data were either parameters of the uniform free-stream or a non-uniform flow field obtained previously for other values of  $M_\infty$  and  $\alpha$ . The turbulence level in the free-stream was set to 1%.

Solutions of the RANS equations were obtained with ANSYS CFX-13 finite-volume solver based on a high-resolution scheme by Barth and Jespersen [4]. An implicit second-order accurate backward Euler scheme was employed for the time-stepping. We used the standard  $k - \omega$  SST and Baseline (BSL) Reynolds stress turbulence models. Computations were performed on hybrid unstructured meshes, which were clustered in the boundary layers, in the wake, and in vicinities of the shock waves. The non-dimensional thickness  $y^+$  of the first mesh layer on the airfoil was about 1.

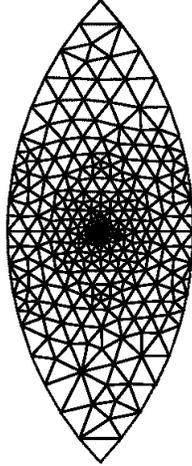


Figure 1: Sketch of the computational domain and mesh.

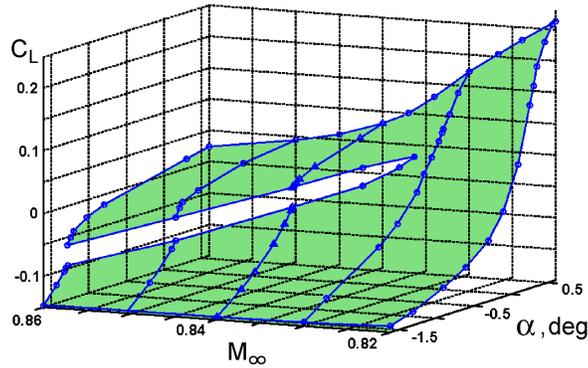


Figure 2: Lift coefficient as a function of the angle of attack and free-stream Mach number for turbulent flow over J-78 airfoil. Computations using the  $k - \omega$  SST model on a mesh of 135,078 cells at  $Re = 5.7 \times 10^6$ .

Verifications of the solver included both analysis of convergence with a mesh refinement and a comparison of test computations for a few benchmark transonic problems with available experimental and numerical data (see details in [5]).

### 3 J-78 Airfoil

Jameson's J-78 airfoil [1] is characterized by a small curvature of the upper surface in the midchord region. At  $p_\infty = 26,400$  Pa,  $T_\infty = 220$  K, and the airfoil chord length  $L = 1$  m, computations using the  $k - \omega$  SST turbulence model [6] yielded a dependence of  $C_L$  on  $M_\infty$  and  $\alpha$  that can be illustrated by a surface displayed in Fig. 2. As seen, the surface involves a slit at  $0.832 \leq M_\infty < 1$ . The slit and jumps of the lift coefficient are caused by the coalescence/rupture of local supersonic regions on the upper surface of airfoil. Figures 3a and 3b illustrate different flows obtained at  $M_\infty = 0.84$  when  $\alpha$  had been approaching the value of  $-0.64$  deg from below and above, respectively.

Figure 4 shows a comparison of the lift coefficients obtained with the CFX-13 solver for the free-stream Mach numbers 0.82 and 0.84 using different turbulence models and two chord lengths  $L$ . In the case  $L = 0.5$  m, the pressure  $p_\infty$  was doubled in order to get the same Reynolds number. The BSL Reynolds stress model at  $L = 1$  m,  $M_\infty = 0.84$  resulted in a continuous plot in contrast to  $k - \omega$  SST model and the BSL model at  $L = 0.5$  m (see Fig. 4b). This discrepancy will be discussed in a subsequent paper by the authors.

At  $M_\infty = 0.82$ , the abrupt rise of  $C_L$  with increasing  $\alpha$  from  $-0.2$  deg to  $0.35$  deg (see Fig. 4a) is explained by both pressure rise on the lower surface and pressure drop on the airfoil's upper surface due to

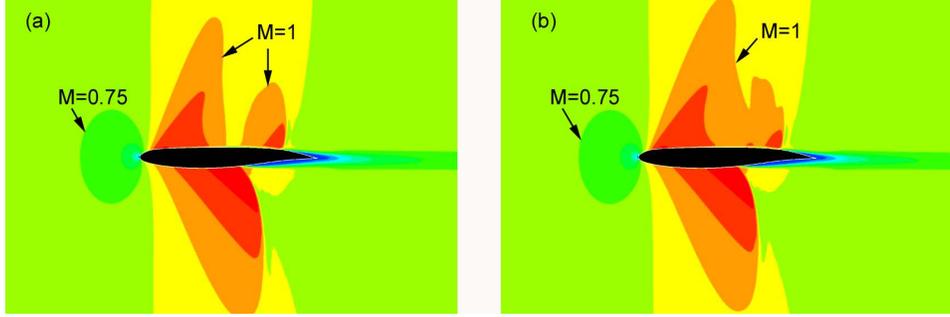


Figure 3: Mach number contours in turbulent flow over J-78 airfoil at  $M_\infty = 0.84$ ,  $Re = 5.7 \times 10^6$ ,  $L = 0.5$  m,  $k - \omega$  SST turbulence model: (a)  $\alpha = -0.64 - 0$  deg, (b)  $\alpha = -0.64 + 0$  deg.

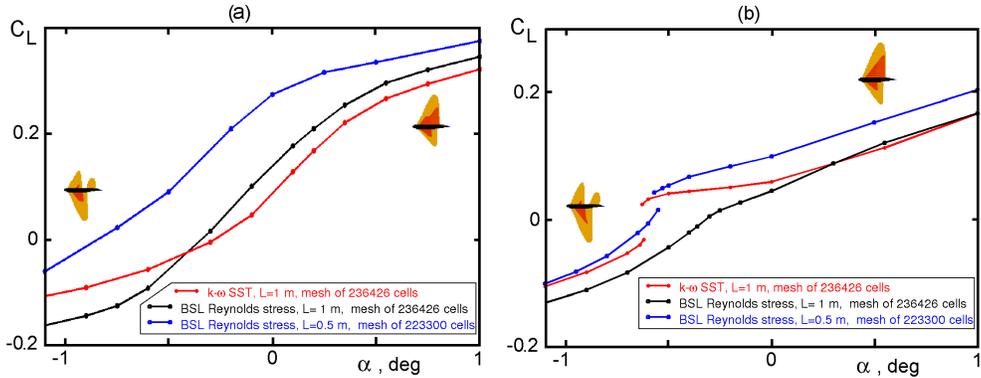


Figure 4: Lift coefficient as a function of the angle of attack for J-78 airfoil calculated at  $Re = 5.7 \times 10^6$ ,  $T_\infty = 220$  K using two turbulent models and two lengths  $L$  of the airfoil chord: (a)  $M_\infty = 0.82$  ( $U_\infty = 243.9$  m/s), (b)  $M_\infty = 0.84$  ( $U_\infty = 249.8$  m/s).

the coalescence of local supersonic regions.

We notice that the free-stream velocity is 243.9 m/s and 249.8 m/s at  $M_\infty = 0.82$  and  $M_\infty = 0.84$ , respectively, as the sound speed is  $\sqrt{(c_p - c_v)c_p T_\infty / c_v} = 297.39$  m/s.

## 4 Boeing 737 Outboard Airfoil

This airfoil determines a mid-span section of the Boeing 737 wing [7]. Figure 5 shows the calculated lift coefficient as a function of the Mach number  $M_\infty$  and free-stream velocity  $U_\infty$  at four angles of attack and  $p_\infty = 54,000$  Pa,  $L = 0.5$  m,  $T_\infty = 250$  K,  $a_\infty = 317.02$  m/s. As seen, the lift coefficient drops crucially when the angle of attack becomes less than  $-1$  deg. This is explained by the concavity of the lower surface of the airfoil near its nose. The concavity provokes formation of a double supersonic region (see Fig. 6), which rapidly expands with decreasing  $\alpha$ . As a consequence, the static pressure on the lower surface drops, and so does the lift coefficient.

It follows from Fig. 5 that the Mach numbers  $M_\infty > 0.80$  are unfavorable for a transition from the cruise flight of the airplane to a descending one. Indeed, for instance, at  $M_\infty = 0.81$  ( $U_\infty = 257$  m/s) if the transition implies a reduction of the angle of attack  $\alpha$  from 1 deg to 0, then a vertical gust of  $-6$  m/s can further reduce the angle  $\alpha$  to  $-1.3$  deg. This will result in a drop of  $C_L$  to zero (see the arrow in Fig. 5) and lead to unsafe conditions for the crew and passengers of the plane.

Also, a drop of  $C_L$  to zero can be caused by a joint effect of a weaker vertical gust of  $-4.5$  m/s, which decreases the angle  $\alpha$  from 0 to  $-1$  deg, and a horizontal gust of 6 m/s, which shifts the efficient free-stream velocity from 257 m/s to 263 m/s. Therefore a vortex enhancing the  $x$ -component of free-stream velocity

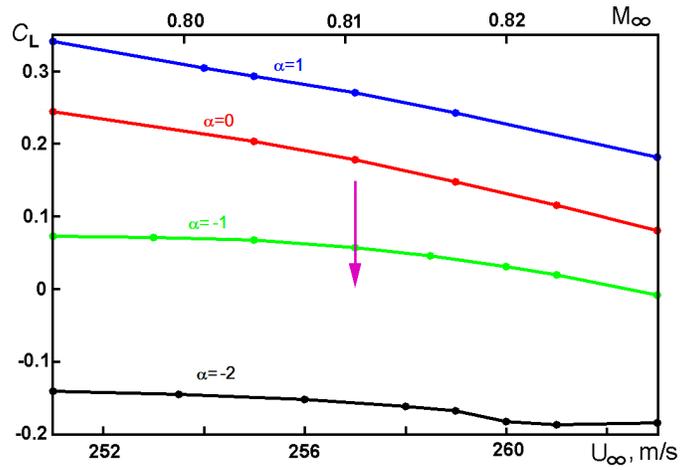


Figure 5: Lift coefficient versus the free-stream Mach number  $M_\infty$  and velocity  $U_\infty$  for the Boeing 737 Outboard airfoil. Calculations using the  $k - \omega$  SST turbulence model at  $Re = 5.7 \times 10^6$ ,  $T_\infty = 250$  K.

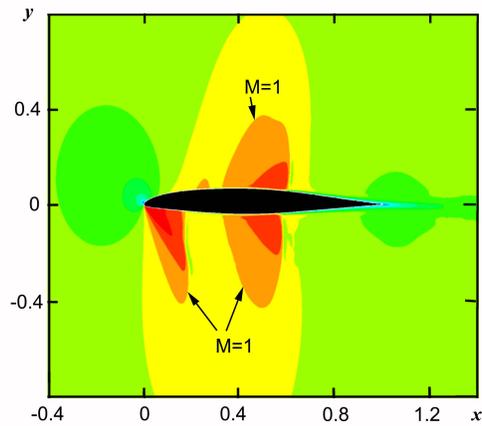


Figure 6: Mach number contours in transonic flow past the Boeing 737 Outboard airfoil at the Mach number  $M_\infty = 0.808$  ( $U_\infty = 256$  m/s), the angle of attack  $\alpha = -2$  deg, and  $Re = 5.7 \times 10^6$ .

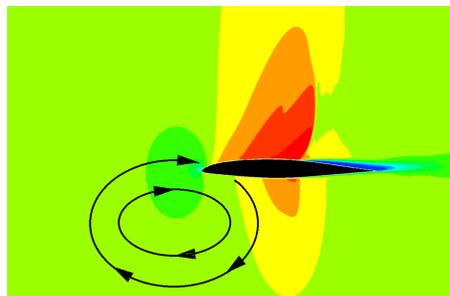


Figure 7: Sketch of an adverse free-stream vortex for the Boeing 737 Outboard airfoil in cruise conditions.

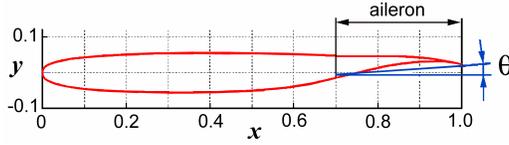


Figure 8: Sketch of the Whitcomb airfoil with an aileron deflected at an angle  $\theta$ .

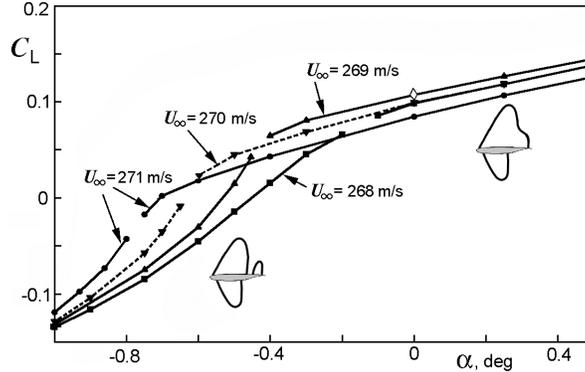


Figure 9: Lift coefficient as a function of the angle of attack  $\alpha$  for transonic flow past airfoil (1) at the aileron deflection angle  $\theta = 4$  deg,  $Re = 5.1 \times 10^6$ . Computations using the  $k - \omega$  SST turbulence model.

and reducing the  $y$ -component (see Fig. 7) is most adverse for the Boeing 737 Outboard airfoil in cruise or descending flight.

## 5 Whitcomb Airfoil with a Deflected Aileron

The last example concerns a Whitcomb airfoil whose rear is modified as follows:

$$y(x) = y_{\text{whitc}}(x) + (x - 0.7) \tan \theta \quad \text{at} \quad 0.7 \leq x \leq 1, \quad (1)$$

where  $y_{\text{whitc}}(x)$  are coordinates of the original Whitcomb airfoil [4]. The modification (1) simulates an aileron deflection upward at the angle  $\theta$  (see Fig. 8). A numerical study of transonic flow over the airfoil (1) was performed at  $p_\infty = 50,000$  Pa,  $L = 0.5$  m,  $T_\infty = 250$  K,  $Re = 5.1 \times 10^6$ .

Figure 9 shows the lift coefficient  $C_L(\alpha)$  obtained for the aileron deflection angle  $\theta = 4$  deg and four free-stream velocities. As seen, the velocity  $U_\infty = 269$  m/s ( $M_\infty = 0.8485$ ) is most adverse from the viewpoint of an airfoil response to variations of the angle  $\alpha$ . At this velocity, a decrease of the angle  $\alpha$  from 0 to  $-1$  deg due to a vertical gust of  $-4.7$  m/s results in a drop of  $C_L$  from 0.1 to  $-0.1$ . The drop is caused by a rupture of the local supersonic region on the upper surface of the airfoil. Figure 10 shows a double supersonic region on the upper surface at  $\alpha = -0.5$  deg.

On the other hand, in certain free-stream conditions, the airfoil response to variations of the deflection angle  $\theta$  may be anomalously small. Figure 11 demonstrates plots  $C_L(\alpha)$  obtained for two positions of the aileron and the velocity  $U_\infty = 269$  m/s. It can be seen that at  $-0.5 < \alpha, \text{deg} < 0.5$ , a change of the aileron position from  $\theta = 0$  to  $\theta = 4$  deg influences  $C_L$  insignificantly. This is explained by a rapid rise of the local supersonic regions on the upper surface of the airfoil. Therefore, at  $U_\infty = 269$  m/s, the aileron fails to properly control the flight. This conclusion is confirmed by plots of lift coefficient versus the deflection angle  $\theta$  at two free-stream velocities (see Fig. 12).

Computations demonstrated that the lift coefficient changes insignificantly when the Reynolds number increases from  $5.1 \times 10^6$  to  $1.5 \times 10^7$ .

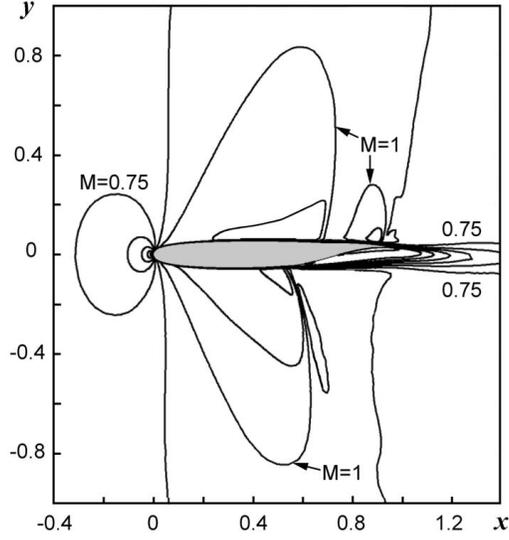


Figure 10: Mach number contours in transonic flow past airfoil (1) at  $\theta = 4$  deg,  $\alpha = -0.5$  deg,  $M_\infty = 0.8485$  ( $U_\infty = 269$  m/s),  $Re = 5.1 \times 10^6$ .

## 6 Conclusion

For the airfoils examined, there exist adverse free-stream conditions that admit crucial changes of the lift coefficient at small variations of the Mach number  $M_\infty$  or the angle of attack  $\alpha$ . At the same time, there exist free-stream conditions in which a response of the Whitcomb airfoil to aileron deflections is anomalously small. In contrast to symmetric airfoils, computations did not reveal oscillations of separated boundary layers at the considered small angles of attack.

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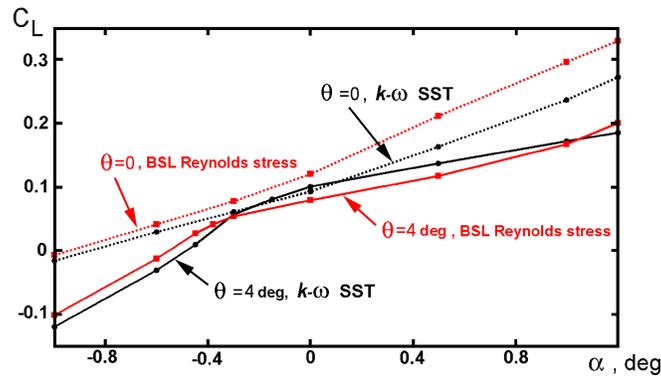


Figure 11: Lift coefficient versus angle of attack for transonic flow over the airfoil (1) at  $M_\infty = 0.8485$ ,  $Re = 5.1 \times 10^6$ .

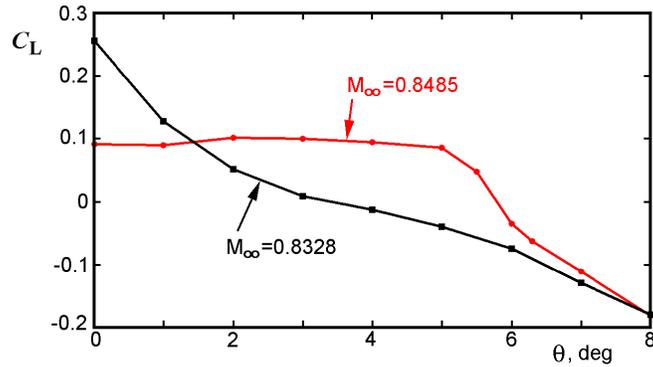


Figure 12: Lift coefficient versus the aileron deflection angle  $\theta$  for transonic flow over the airfoil (1) at  $\alpha = 0$ ,  $Re = 5.1 \times 10^6$ . Computations using the  $k - \omega$  SST turbulence model.

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