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A near-wall model for heat transfer at high Prandtl numbers

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Abstract: Main drawback of the Reynolds-averaged Navier-Stokes (RANS) predictions of heat transfer at elevated Prandtl numbers is lack of accuracy, accompanied with increased mesh sensitivity. Even in conjunction with an accurate near-wall turbulence modeling, majority of the wall heat transfer models (based on the so-called P functions) fails to capture heat transport in the diffusive sub-layer and the thermal buffer layer. This is primarily due to variation of the turbulent Prandtl number (Pr_t) in the wall proximity that is not properly accounted for. Applicability of the temperature wall functions is generally restricted to the log-law region, but can be partially extended to the near-wall region by means of various hybrid strategies. They utilize blending functions dependent on the molecular Prandtl number, which do not necessarily provide an adequate behavior in the thermal buffer layer. Based on the direct numerical simulations (DNS) data for the heated pipe (Re_τ=360, Pr=1-20) and channel (Re_τ=150, Pr=100-500) flows, a two-layer formulation suitable for implementation in the RANS framework has been developed. The new model, denoted as PRTL - Prandtl Thermal-Layer model, improves predictions of non-dimensional temperature profiles close to the wall, exhibiting significantly reduced mesh sensitivity for 1<Pr<2900.

Keywords: high Pr number flows, thermal layer, near-wall heat transfer

1 Introduction

A new era of vehicle electrification is going to pose a number of issues related to cooling of various electric and hybrid powertrain components. Their proper cooling is of decisive importance for reliable performance under various operating conditions. Design of a battery cell, pack or system has to ensure that the battery never reaches the thermal runaway temperature, which irreversibly triggers a chain reaction of self-heating and ultimately destruction of the cell [1]. Predictive computational fluid dynamics (CFD) simulations can be used to understand and quantify aspects of thermal behavior of various components of the battery thermal management systems, leading to improved and optimized design solutions. This kind of industrial applications is challenging for CFD as liquid coolant flow properties give rise to a wide range of molecular Prandtl numbers (mixtures of ethylene glycol and water-moderate, oils-high Pr numbers), at the same time being associated with relatively low Reynolds numbers. In these flow configurations, RANS calculations are expected to suffer from a lack of accuracy and/or increased mesh dependency. This might be attributed to an improper near-wall turbulence modeling, but mostly it is due to the deficiency of the wall heat transfer models (based on the so called P-functions) that do not properly account for the variation of the turbulent Prandtl number in the wall proximity [2,3,4]. As the diffusive sub-layer gets significantly thinner than the viscous velocity sub-layer (for Pr >1), treatment of the thermal buffer layer gains importance as well. Recently, Irrenfried and Steiner [3] reported on the extensive DNS study of a uniformly heated pipe flow at Re_τ=360, featuring Prandtl numbers 1<Pr<20. They proposed a significantly improved P-function modeling concept that is applicable irrespective of the non-dimensional wall distance (y^+). In terms of

implementation and robustness, a model that requires numerical integration is not appealing for the majority of the CFD codes. Utilizing the same DNS database, a simpler two-layer model concept was presented by Šarić et al. [5], however, its validation and applicability is limited to a narrow Prandtl number range excluding oils as frequently used liquid coolants. Based on the additional DNS data for a heated channel flow (Re_τ=150, Pr=100-500) provided by Bergant and Tiselj [6], this paper presents formulation and validation of a new near-wall model for heat transfer at high Prandtl numbers (1<Pr<2900), denoted as Prandtl Thermal-Layer (PRTL) model.

2 Underlying Turbulence Model

The present heat transfer modeling relies on the k-ζ-f turbulence model with the hybrid wall treatment, which is capable of predicting the near-wall momentum and heat transfer with more fidelity, compared to the standard or low-Re variants of the k-ε turbulence model [7]. Irrespective of complexity of the heat transfer model, its performance strongly relies on capability of the underlying turbulence model to capture near-wall transport phenomena. The k-ζ-f RANS model employed in this work relies on the elliptic relaxation concept, thus avoiding the need for any wall topology parameter and damping functions. The variable ζ represents the ratio v^2/k (is a scalar property in the Durbin's v^2-k model [8], which reduces to the wall-normal stress in the near-wall region) providing more convenient formulation of the equation for ζ and especially of the wall boundary condition for the elliptic function f. Hanjalić et al. [9] demonstrated that the model is numerically very robust and more accurate compared to the simpler two-equation eddy viscosity models. The equations constituting the k-ζ-f model read:

$$\frac{\partial k}{\partial t}+U_j\frac{\partial k}{\partial x_j}= P-\varepsilon +\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] \tag{1}$$

$$\frac{\partial \varepsilon}{\partial t}+U_j\frac{\partial \varepsilon}{\partial x_j}=\frac{C_{\varepsilon 1}P-C_{\varepsilon 2}\varepsilon}{T}+\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] \tag{2}$$

$$\frac{\partial \zeta}{\partial t}+U_j\frac{\partial \zeta}{\partial x_j}= f-\frac{\zeta}{k}P+\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_t}{\sigma_\zeta}\right)\frac{\partial \zeta}{\partial x_j}\right] \tag{3}$$

$$L^2\nabla^2 f-f=\frac{1}{T}\left(C_1+C_2\frac{P}{\varepsilon}\right)\left(\zeta-\frac{2}{3}\right) \tag{4}$$

with the wall boundary condition for f:

$$f_{wall}=\lim_{y\to 0}\left(-2\nu\zeta/y^2\right) \tag{5}$$

Here, T represents a switch between the turbulent time scale $\tau=k/\varepsilon$ and the Kolmogorov time scale $\tau_k=(\nu/\varepsilon)^{1/2}$:

$$T=\max\left[\frac{k}{\varepsilon},C_\tau\left(\frac{\nu}{\varepsilon}\right)^{1/2}\right] \tag{6}$$

The corresponding length scale L is obtained as a switch between the turbulent and Kolmogorov length scales:

$$L=C_L\max\left[\frac{k^{3/2}}{\varepsilon},C_\eta\left(\frac{\nu^3}{\varepsilon}\right)^{1/4}\right] \tag{7}$$

Values of the coefficients appearing in the model equations are outlined in Table 1. The hybrid wall treatment employed here blends the integration up to the wall (exact boundary conditions) with the

high-Reynolds number wall functions (for the wall shear stress, production and dissipation of the turbulent kinetic energy), enabling well-defined boundary conditions irrespective of the position of the wall-closest computational node. This approach ensures numerical robustness which is required in the industrial CFD simulations.

Table 1: Model coefficients

C_1	C_2	$C_{\varepsilon 1}$	$C_{\varepsilon 2}$	C_μ	
0.4	0.65	$1.4(1+0.045/\zeta^{0.5})$	1.9	0.22	
σ_k	σ_ε	σ_ζ	C_τ	C_L	C_η
1.0	1.3	1.2	6.0	0.36	85

More specific details about the model developments can be found in the reference publications [7,9,10]. It is important to note that the underlying hybridized turbulent quantities are used in their original form, aiming at a DNS-based improvement of near-wall heat transfer modeling. Furthermore, the hybrid heat transfer model based on the Jayatilleke temperature wall function will serve as a reference for evaluation of the model performance.

3 Heat Transfer Modeling

Even in conjunction with an accurate near-wall turbulence modeling, which correctly predicts the momentum transfer and the turbulent viscosity (ν_t) profile near the wall ($y^+<5$), majority of the wall heat transfer models (based on the so-called P functions) fails to capture heat transport in the diffusive sub-layer and the thermal buffer layer. This is primarily due to variation of the turbulent Prandtl number (Pr_t) in the wall proximity, which is not properly modeled/accounted for (cf. Figure 1).

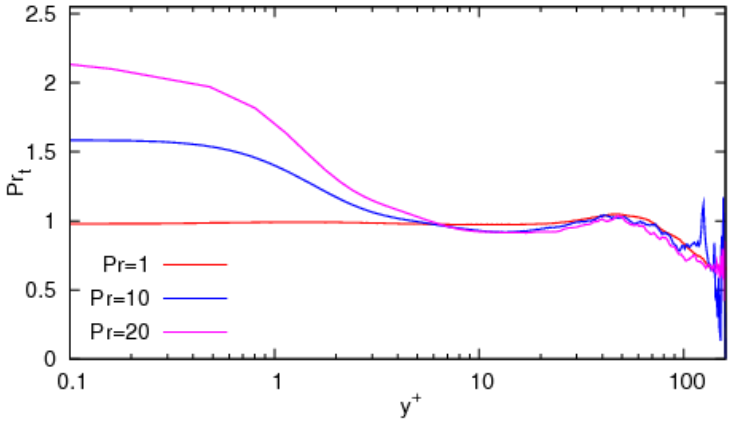

Figure 1: . Variation of the turbulent Prandtl number obtained from the DNS data [5]

The temperature wall function used in most of the CFD codes is defined assuming a constant turbulent Prandtl number (Pr_t =0.9) as follows:

$$T_{\log}^{+} = \mathrm{Pr}_t\left(\frac{1}{\kappa}\ln Ey^{+} + P\right) \tag{8}$$

where P-function proposed by Jayatilleke [11] is adopted in the present work:

$$P = 9.24\left[\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_t}\right)^{0.75} - 1\right]\left[1 + 0.28e^{-0.007\frac{\mathrm{Pr}}{\mathrm{Pr}_t}}\right] \tag{9}$$

It relies on numerous experimental data, being sensitive to variations of the molecular Prandtl number. The reference, hybrid wall heat transfer model using the Jayatilleke temperature wall function as the upper bound takes the following form:

$$T^+_{hyb} = \mathrm{Pr}\, y^+ e^{-\Gamma} + \mathrm{Pr}_t\left(\frac{1}{\kappa}\ln Ey^+ + P\right)e^{-\frac{1}{\Gamma}} \quad , \quad \Gamma = 0.01\frac{(\mathrm{Pr}\, y^+)^4}{1+5\mathrm{Pr}^3 y^+} \qquad (10)$$

with Γ denoting the originally proposed blending function of Kader [12]. Predictive capability of the k-ζ-f turbulence model is illustrated by Figure 2, which compares the predicted dimensionless temperature profiles against the DNS data. A turbulent, uniformly heated pipe flow (Re=5300) at various Prandtl numbers is computed by means of RANS using a simple, practically axisymmetric configuration with the fine numerical grid (y_{wall}^+=0.2). Constant fluid properties and boundary conditions are specified according to the reference DNS [3]. Provided that a fine mesh is used, the RANS model is evidently capable of capturing the temperature distribution for the considered Prandtl numbers (Pr=1,10 and 20). The hybrid non-dimensional temperature reduces to T^+_{lam}=Pr y^+ so that the calculated wall heat transfer relies purely on the predicted momentum and turbulent transport, demonstrating accuracy of the presently used near-wall turbulence model. Along with the computed profiles, the hybridized Jayatilleke model (T^+_{hyb} JT) is depicted as the reference for a priori assessment. Performance of the hybrid model is expected to deteriorate, in particular if the first numerical grid point falls around y^+=2 and y^+=8, where over-predictions and under-predictions of the wall heat transfer, respectively, are expected. With further increase in the Prandtl number, the diffusion sub-layer gets substantially thinner than the viscous velocity sub-layer and such a fine mesh resolution becomes unaffordable. Consequently, the first near-wall computational node is likely to reside in the thermal buffer layer or within the viscous-thermal buffer layer, necessitating an improved modeling of heat transport in this part of the thermal layer.

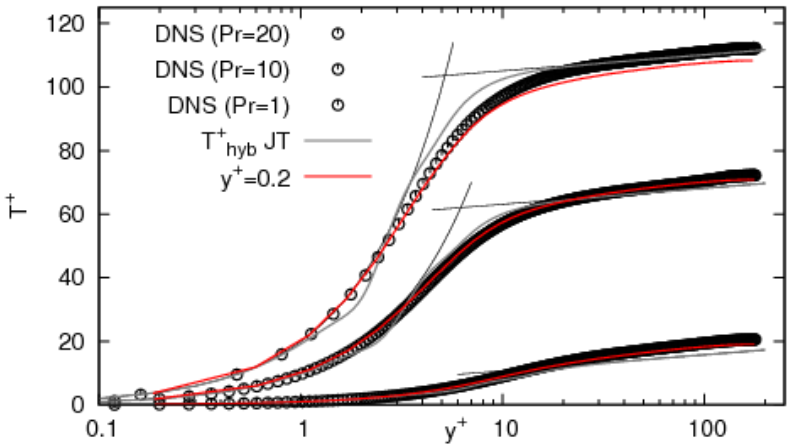

Figure 2: . Predicted T^+ evolutions using the fine mesh for Pr=1,10 and 20 [3,5]

3.1 Prandtl Thermal-Layer Model

The P-function based modeling concept ($T^+ = \mathrm{Pr}_t[U^+ + P]$) has been recently revisited by Irrenfried and Steiner [3]. They conducted an extensive study of a uniformly heated pipe flow featuring moderate Prandtl numbers (1-20) by means of DNS. The authors proposed a significantly improved *P*-function modeling concept, being applicable irrespective of the non-dimensional wall distance y^+. This model, derived using comprehensive DNS data for a turbulent heated pipe flow at Pr=1,10 and 20, and Re=5300/Re_τ=360 (based on the pipe diameter), is briefly outlined by the following set of equations:

$$T^+ = \mathrm{Pr}_{t,\infty}\left(U^+ + P\right) \ , \ U^+ = \frac{1}{\kappa}\ln y^+ + \beta \tag{11}$$

$$P = \int_0^{U^+} \left(\frac{1+\frac{\nu_t}{\nu}}{\frac{\mathrm{Pr}_{t,\infty}}{\mathrm{Pr}} + \frac{\mathrm{Pr}_{t,\infty}}{\mathrm{Pr}_t}\frac{\nu_t}{\nu}} - 1 \right) dU^+ \tag{12}$$

$$\frac{dU^+}{dy^+} = \frac{2}{1+\sqrt{1+4l_m^{+2}}} \tag{13}$$

with $\mathrm{Pr}_{t\infty}$=0.9, κ=0.34, β=4.5 and l_m^+ representing van Driest's mixing length. As described in [3], using the DNS-based, enhanced model for turbulent Prandtl number, equation (12) can be numerically integrated to obtain the distribution of $P(U^+)$. A model that requires numerical integration is not practical for the usage in the industrial CFD applications. Utilizing the same DNS database, a simpler two-layer concept was presented by Šarić et al. [5], with the numerically integrated T^+ model serving as an additional reference to ensure comparable accuracy. In absence of the reliable data for elevated Prantdl numbers (e.g. oils), the initial formulation is valid for a limited Pr number range, being applicable to flows involving the mixtures of ethylene glycol and water. Considering the extended database with the most accurate results presently available (so-called "under-resolved" DNS – approach based on the DNS of the velocity field and LES-like treatment of the temperature field), a new near-wall model for heat transfer at high Prandtl numbers, denoted as Prandtl Thermal-Layer (PRTL) model will be outlined. We would like to express our gratitude to the authors (Bergant and Tiselj [6]) for sharing the results of their simulation of the turbulent heat transfer in the channel flow (Re_τ=150, Pr=100,200 and 500).

Neglecting the tangential derivatives, pressure gradient and assuming the constant thermal properties, the simplified boundary-layer equation for energy is expressed as follows:

$$-\frac{\rho c_p u^*}{q_w} dT = \frac{1}{\left(\frac{1}{\mathrm{Pr}} + \frac{\nu^+}{\mathrm{Pr}_t}\right)} dy^+ \tag{14}$$

with

$$y^+ = \frac{y u^*}{\nu} \ , \ \nu^+ = \frac{\nu_t}{\nu} \tag{15}$$

It is essential to provide a suitable model of T^+ in the vicinity of the wall (incorporating the Pr_t variation shown in Figure 1) by means of analytical integration of the above equation rearranged as:

$$dT^+ = \frac{1}{\left(\frac{1}{\mathrm{Pr}} + \frac{1}{\mathrm{Pr}_t}\frac{\nu_t}{\nu}\right)} dy^+ \tag{16}$$

In line with the derivation of the model of Han and Reitz [13] and based on the present DNS data, a simplified expression describing the variation of turbulent viscosity, in conjunction with turbulent Prandtl number, is used to integrate the right-hand side of eqn. (16):

$$\frac{1}{\mathrm{Pr}_t}\frac{\nu_t}{\nu} = a + by^+ + cy^{+^2} \qquad y^+ < y_{TL}^+ \tag{17}$$

with the constants a,b,c to be determined from the available database and y_{TL}^{+} =40 denoting the limiting value. Thereafter, a smooth transition from the thermal layer to the thermal log-law region is ensured by equating a properly modified logarithmic $T_{\log}^{+}$ profile to the corresponding near-wall value:

$$T_{\log}^{+}(y_{TL}^{+}) = T_{nw}^{+}(y_{TL}^{+}) \tag{18}$$

Recall that Jayatilleke P-function [11] is adopted as the upper bound introducing a simple correction factor f_{TL}. The two-layer model then takes the following form:

$$T_{nw}^{+} = \int_{0}^{y_{TL}^{+}} \frac{1}{\mathrm{Pr}^{-1} + a + by^{+} + cy^{+2}}\, dy^{+} = \frac{2}{\sqrt{4c\left(\mathrm{Pr}^{-1}+a\right) - b^{2}}} \arctan\left(\frac{2cy^{+} + b}{\sqrt{4c\left(\mathrm{Pr}^{-1}+a\right) - b^{2}}} \right)\Bigg|_{0}^{y_{TL}^{+}} \tag{19}$$

for $y^{+} \leq 40$, with the modified logarithmic profile in the outer layer ($y^{+}>40$):

$$T_{\log}^{+} = Pr_{t}(U^{+} + f_{TL}P) \tag{20}$$

Note that due to the assumed second order polynomial fit (Eq. 17), the model is limited to the Prandtl numbers that permit positive values of the integrand (Eq. 19). Nevertheless, this has a negligible impact on the final integral values. The model constants are summarized in Table 2. Interestingly, the correction factor f_{TL}, which is introduced to impose continuity of a two-layer approach, is close to unity for Pr>20, indicating that the original log-law profile of the Jayatilleke temperature wall function is preserved. The prescribed model constants a,b and c are not strictly function of the Prandtl number, but are determined for the two distinct ranges of Prandtl numbers. Despite this relaxing assumption, dependency of the integrand on Pr number is retained yielding acceptable agreement with the available DNS database. Due to aforementioned limitation of the integrand (positive values), the present model is applicable for Pr<2907. Figure 3 displays a priori evaluation of the resulting two-layer formulation against the reference (a hybrid version of Jayatilleke model) and the available DNS results for Pr=1-20 [3]. Additional comparison for the channel flow at significantly higher Prandtl numbers [6] is shown in Figure 4. Evidently, the PRTL model improves non-dimensional temperature profiles in the thermal buffer layer for the range of the investigated Pr-numbers.

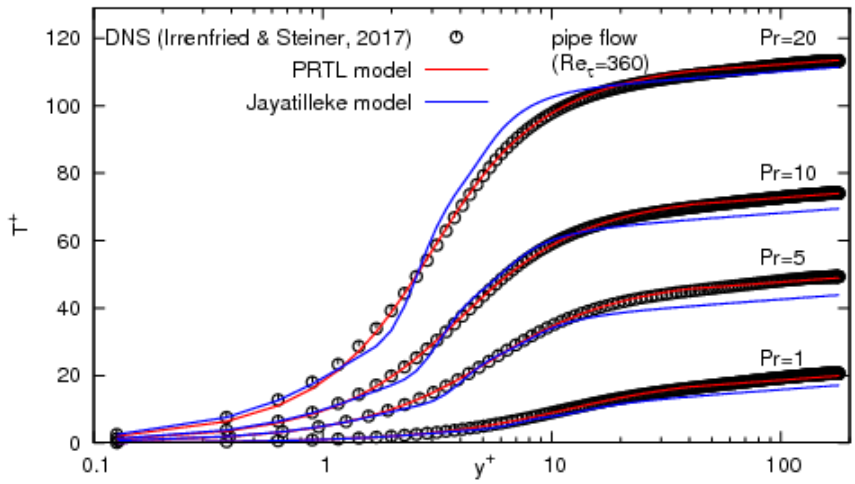

Figure 3: A priori evaluation of the new model against the reference data for Pr=1-20 [3]

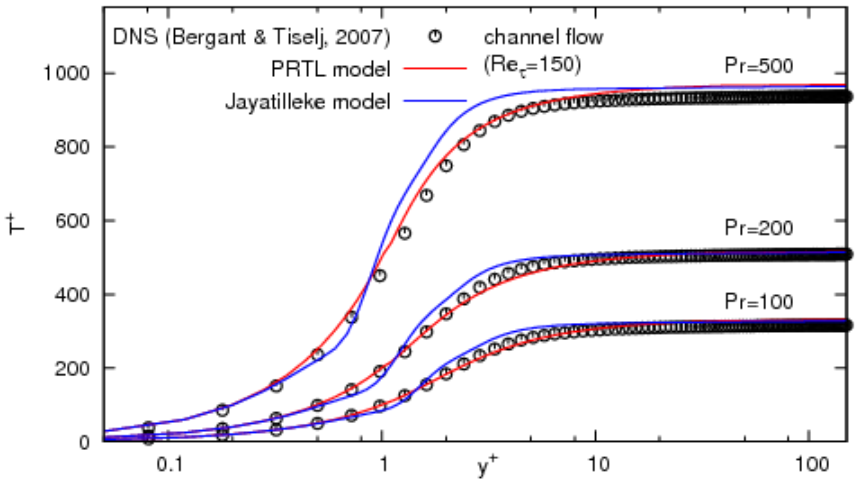

Figure 4: A priori evaluation of the new model against the reference data for Pr=1-500 [6]

Table 2: Summary of the PRTL model constants

Pr	1	5	10	20	100	200	500
f_{TL}	4.22	1.185	1.085	1.022	1.011	1.003	1.005
a	0.0137	0 < Pr < 50			0.0017	50 ≤ Pr < 2907	
b	-0.0213				-0.0055		
c	0.0071				0.0037		

3.2 Results

After a priori assessment of the proposed model, simulations of the relevant heated pipe flows and a real case representing cooling jacket of an e-motor will be presented. One peculiarity pertinent to the model implementation deserves a special attention. In order to fully exploit its potential benefits, y^+ has to be calculated either from the already hybridized wall shear stress:

$$y^+ = \frac{y_p\sqrt{\frac{\tau_w}{\rho}}}{\nu} \quad , \qquad \tau_w = \mu_w \frac{U_p}{y_p} \tag{21}$$

with

$$\mu_w = \mu \frac{U_p^*}{y_p^*} \quad U_p^* = \frac{1}{\kappa}\ln E y_p^* \quad , \quad y^* = \frac{c_\mu^{1/4} k^{1/2} y_p}{\nu} \tag{22}$$

or alternatively, based on the mean/total friction velocity defined as:

$$u_\tau = \sqrt{(u_{\tau,\,lam})^2 + (u_{\tau,\,tur})^2} \tag{23}$$

The latter expression is found to be more suitable for the entire Pr-number range. The RANS calculations of a heated pipe flow were conducted using several numerical grids. The representative results for Pr=10 are shown in Figure 5. As one can observe, the results obtained by the reference model (T^+_{hyb} JT) reveal some deficiencies in the thermal buffer layer that were already anticipated by the foregoing a priori analysis. On the other hand, the PRTL results corroborate enhancements in the predicted dimensionless temperature profiles, exhibiting excellent agreement with the reference DNS

data. One can expect that variable fluid properties induce modifications of local Reynolds and Prandtl numbers in the wall proximity, which are conflicting the underlying assumptions of the wall function approach. In order to evaluate the new model in case of slightly higher Reynolds number flow featuring variable thermal properties, the heated pipe flow with a water/glycol mixture as working fluid is simulated. The results presented in Figure 6 prove superiority of the PRTL model which features substantially decreased mesh dependency.

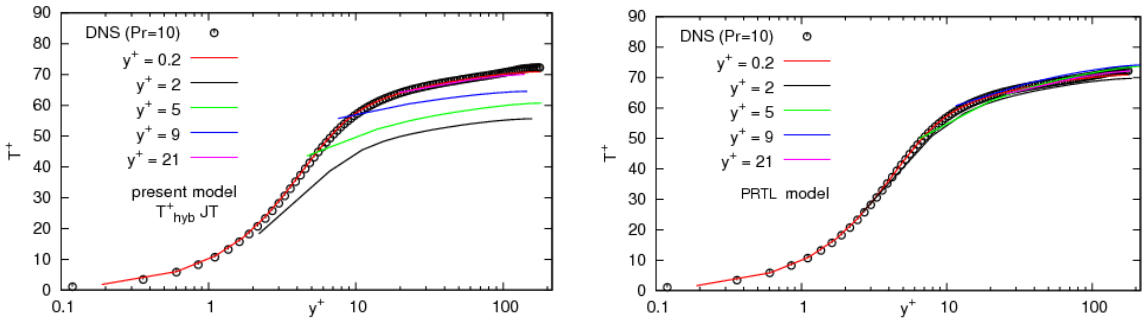

Figure 5: Improvement of RANS predictions using the PRTL model (heated pipe flow at Pr=10[3])

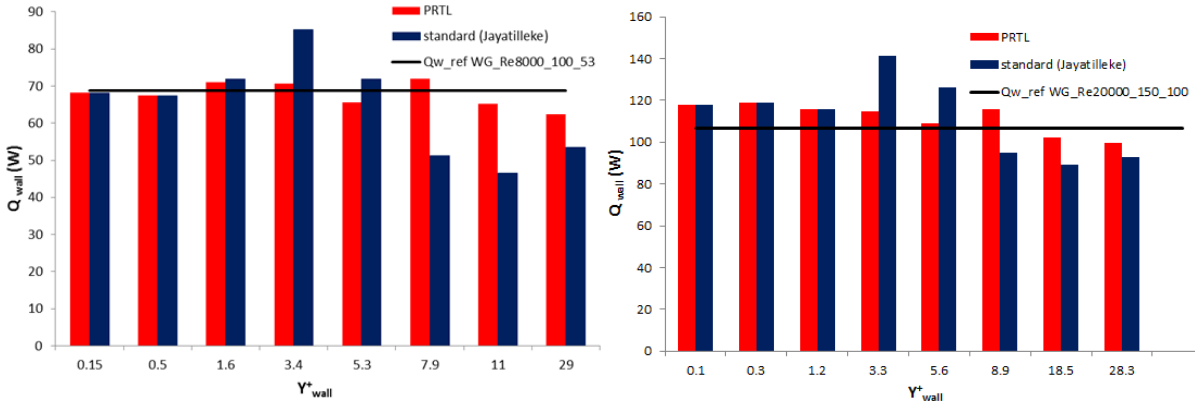

Figure 6: Predicted Q_{wall} in a heated pipe flow (50% water/glycol mixture)
Re=8000, Pr=6-15 (left), Re=8000, Pr=4-6 (right), with $Q_{w\text{-}ref}$ from Gnielinski correlation [14]

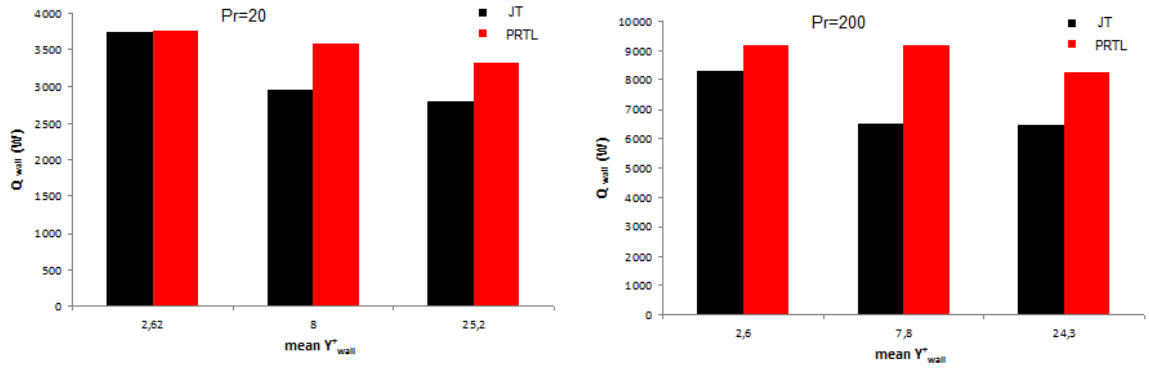

Figure 7: Predicted heat rates for various mesh resolutions (E-motor cooling jacket, Pr=20 and 200)

The model performance in practical applications is illustrated by simulation of a real case representing cooling jacket of an e-motor. Reynolds numbers in the channels are comparable to the one in the previously investigated DNS test case. The wall temperature is kept constant and the coolant has constant properties with the molecular Prandtl number Pr=20 and 200, respectively, which encompass both mixtures of ethylene glycol and water and oils. Three different polyhedral meshes used in the

computations are characterized by the mean near-wall y^+ values ranging from 2.6 to 25. Heat transfer predictions obtained by the PRTL and Jayatilleke models are compared in Figure 7, which confirms enhancement with respect to mesh independency for the relevant near-wall resolutions. Note that some discrepancy for the coarsest mesh is expectable for such a low Reynolds number flow ($Re_{channels}\approx5000$).

4 Conlusions

Industrial applications involving liquid coolants are challenging for CFD due to the thermal properties that give rise to a wide range of molecular Prandtl numbers, being typically associated with relatively low flow Reynolds numbers. Even in conjunction with an accurate near-wall turbulence modeling, the present wall heat transfer models fails to capture heat transport in the diffusive sub-layer and the thermal buffer layer. This is primarily due to variation of the turbulent Prandtl number (Pr_t) in the wall proximity (usually taken as a constant Pr_t=0.85-0.9), which is not properly incorporated in the models. Utilizing the extended DNS database with the most accurate results presently available for turbulent heat transfer in a heated pipe/channel, a new RANS near-wall model for heat transfer at high Prandtl numbers, denoted as Prandtl Thermal-Layer (PRTL) model has been be proposed. As illustrated by several relevant benchmark calculations, the model improves predicted wall heat transfer, exhibiting significantly reduced mesh sensitivity that enables high fidelity thermal simulation of complex thermal systems involving a wide range of Prandtl numbers ($1<Pr<2900$). Further enhancements of the presented concept will consider effects of higher Reynolds number. The sub-grid scale Prandtl number behavior close to the wall be assessed as well, aiming at similar application to the sub-grid scale modeling in the framework of large-eddy simulation (LES).

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