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Sensitivity Analysis of Non-linear Steep Waves using VOF Method

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Abstract: The analysis and prediction of non-linear waves is a crucial part of ocean hydrodynamics. Sea waves are typically non-linear in nature, and whilst models exist to predict their behavior, limits exist in their applicability. In practice, as the waves become increasingly steeper, they approach a point beyond which the wave integrity cannot be maintained, and they 'break'. Understanding the limits of available models as waves approach these break conditions can significantly help to improve the accuracy of their potential impact in the field. Moreover, inaccurate modeling of wave kinematics can result in erroneous hydrodynamic forces being predicted.

This paper investigates the sensitivity of non-linear wave modeling from both an analytical and a numerical perspective. Using a Volume of Fluid (VOF) method, coupled with the Open Channel Flow module in ANSYS Fluent, sensitivity studies are performed for a variety of non-linear wave scenarios with high steepness and high relative height. These scenarios are intended to mimic the near-break conditions of the wave. 5^{th} order solitary wave models are applied to shallow wave scenarios with high relative heights, and 5^{th} order Stokes wave models are applied to short gravity waves with high wave steepness. Stokes waves are further applied in the shallow regime at high wave steepness to examine the wave sensitivity under extreme conditions. Comparisons of spatial and transient discretization methods, implicit versus explicit formulations, and time step size are also conducted.

The study concludes that accurate prediction of wave kinematics at near-breaking condition depends on the choice of an appropriate high order wave model, along with appropriate high order free surface numerics. In particular, the cases studied in this paper illustrate the applicability of 5th order wave models in the shallow and deep-water regimes, and the computational benefit of implicit numerical formulations combined with a second order transient approach to achieve accurate results, even at relatively large time step sizes. The practical numerical limits of wave steepness and relative height for each case are also identified using this approach.

Keywords: Non-linear Steep Waves, Computational Fluid Dynamics, Volume of Fluids.

1 Introduction

Sea waves are typically non-linear in nature and their analysis near breaking condition is one of the critical aspects of designing offshore structures and moving vessels because wave energy is significantly dissipated after breaking.

The non-linearity of waves is measured by either wave steepness or relative height depending on the depth of the sea [1]. Figure1A represents a schematic diagram of a wave and its parameters whereas Figure1B shows different wave regimes and wave breaking criteria [2]. Stokes wave theory variants, which are expansion series of wave steepness, are used in deep sea whereas Solitary wave variants, which are expansion series of relative height, are used in the shallow depth. The theoretical limits of wave steepness and relative height without wave-breaking have been shown to be 0.142 and 0.78 [2] and the order of wave theory plays a crucial role in achieving these limits. High order wave theories have been proposed by different authors, [3,4], however, 5th order variants have been found to be accurate for wave steepness up to 0.12 and relative height up to 0.55, which are more than adequate for most practical scenarios [5].

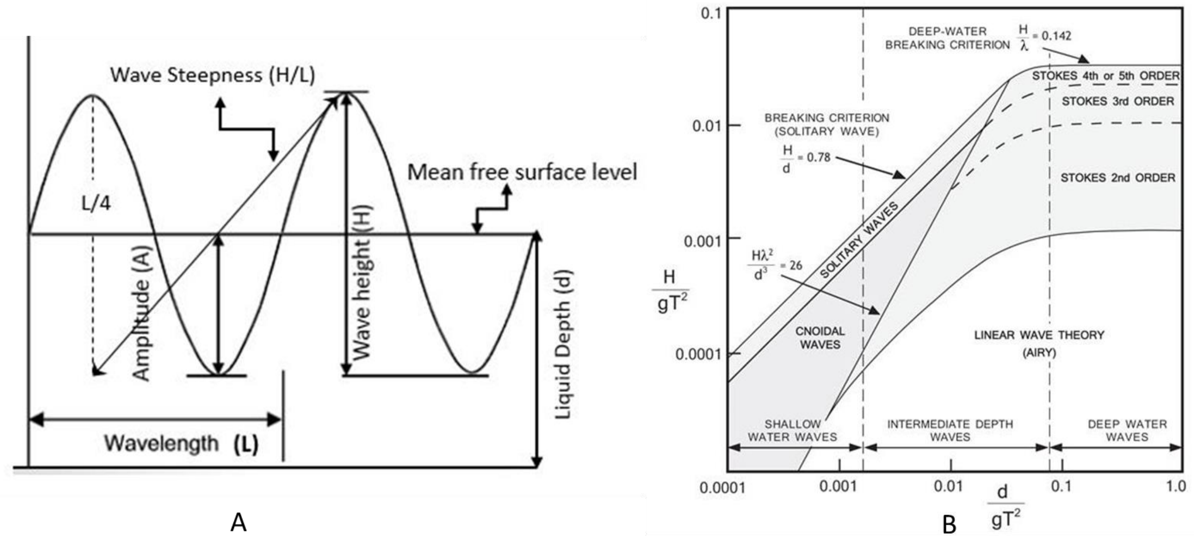


Figure 1 (A): Schematic drawing of wave and its parameters and Figure 1 (B): The limits of validity of wave theories [2]

Volume of Fluid (VOF) is a widely used computational method for capturing of free surfaces in wave simulations. In this method, the choice of explicit or implicit formulation, and the appropriate spatial and transient discretization are key for capturing the physics of non-linear waves just under breaking condition. Explicit formulations tend to be more accurate than implicit formulations, but time step size is limited by Courant based stability criterion. In addition, the solutions can become extremely sensitive to time step size with the increasing non-linearity and thus explicit formulations are generally not a preferred method for wave simulations. Implicit formulations, on the other hand, allow larger time step sizes to be used and exhibit less sensitivity with increasing non-linearity. Implicit formulations are numerically more diffusive than explicit formulations, but this can be compensated for by using higher order time formulation to maintain higher accuracy.

In this study, results obtained with the VOF method using ANSYS Fluent are compared with analytical solutions of higher order wave theory in a variety of contexts. Sensitivity analyses are also conducted to determine the effect of spatial and temporal discretization, and implicit and explicit formulation, on the accuracy of the VOF solution.

In this paper, the first case study examines the sensitivity of a 5th order Solitary wave with different relative heights to explicit and implicit formulations including first and second order transient methods.

The second case study examines the time-step size sensitivity of a Stokes wave with different wave steepness to explicit and implicit formulations including first and second order transient methods. The sensitivity of a Stokes wave at high steepness is further examined by using Skjelbria and Fenton variants. The third case study examines the sensitivity of a Stokes wave at high steepness as well as high relative height.

2 Numerical Methodologies

2.1 Governing Equations

The governing equations are discretized based on Finite Volume Method (FVM). Volume of Fluid Method (VOF) is used to track interfaces between non-penetrating fluids. Volume Fraction of a specific fluid (α) is defined as the ratio of the volume of that fluid to total volume. Interfaces between different fluids are identified by volume fraction falling between 0 and 1.

Summation of volume fraction for all the fluids should be equal to one

$$\sum_{a} \alpha = 1$$

Volume fraction equation is given as,

$$\frac{\partial \alpha}{\partial t} + \nabla \cdot (\vec{u}\alpha) = 0$$

Total continuity equation for incompressible fluid is given as,

$$\nabla \cdot \vec{u} = 0$$

Resulting velocity field is shared among the phases after solving a single momentum equation throughout the domain.

$$\frac{\partial \rho \vec{u}}{\partial t} + \nabla \cdot (\rho \vec{u} \otimes \vec{u}) = -\nabla p + \nabla \cdot \overline{\overline{T}} + \vec{F}_b$$

The properties in the total continuity and momentum equations are volume weighted averaged properties.

2.2 Waves Theories:

2.2.1 Stokes Wave Theory: Stokes wave theories formulated in ANSYS Fluent code are based on the work by John D. Fenton [1]. These wave theories are valid for high steepness finite amplitudes waves operating in intermediate to deep liquid depth range. The generalized expression for wave profile for 5th order Stokes theory is given as,

$$\varsigma(X,t) = \frac{1}{k} \sum_{i=1}^{5} \sum_{j=1}^{i} b_{ij} \xi^{i} \cos(jk(x - ct))$$

Where, $\xi = \frac{kH}{2} = \frac{\pi H}{L}$ and $k = \frac{2\pi}{L}$

$$c = \sqrt{\frac{g}{k} \tanh(kd)} \left(\sum_{i=1}^{5} c_i \xi^{i} \right)$$

Where c is wave celerity and k is wave number, b_{ij}, c_i are complex expressions of kH [5].

2.2.2 Solitary Wave Theory: Solitary wave theories are more widely used for shallow depth regimes. Solitary wave theory expressions are derived by assuming that the waves have infinite wavelength. 5th order solitary wave expressions are complex functions of relative height (H/d) [3,4], therefore first order expressions for solitary wave are given here for simplicity.

Wave profile for a shallow wave is defined as

$$\varsigma(X,t) = H \sec h^2 \left[\frac{k}{d} \left(x - x_0 - ct \right) \right]$$

where wave celerity and wave numbers are given as,

$$c = \sqrt{gd} \left(1 + \frac{H}{2d} \right) \text{ and } k = \frac{1}{d} \sqrt{\frac{3H}{4d}}$$

and x_0 is the initial position of wave.

2.3 Solver Methods

2.3.1 Pressure Velocity Coupling: PISO method is applied for pressure-velocity coupling which is based on solving the pressure correction equation using a predictor and corrector approach. It allows large under-relaxation factors for pressure and momentum which help in achieving faster convergence and is generally recommended for transient flows.

2.3.2 Volume Fraction Formulation: Fully implicit and fully explicit variants are used for formulating volume fraction equation. The Explicit formulation allows both interface tracking and capturing schemes for accurate modeling of interfaces, however, it requires small time step sizes to meet Courant number-based stability criterion. The Implicit formulation allows only interface capturing schemes which are numerically more diffusive compared to interface tracking schemes, however, it allows rather larger time step sizes which is preferable to achieve results quickly.

2.3.3 Volume Fraction Discretization: In the present study, the interface capturing scheme is used in both implicit and explicit formulations. Face values of volume fraction used in the convection term are discretized using the second order reconstruction scheme based on slope limiters [1].

$$\alpha_f = \alpha_d + \beta \nabla \alpha_d . dr$$

Where, α_f is face volume fraction, α_d is donor cell volume fraction, $\nabla \alpha_d$ is donor ell volume fraction gradient, β is slope limiter and dr is position vector between cell to face centroid.

2.3.4 Time Formulation: Transient terms are discretized either by first order or second order methods. The bounded second order time implicit method ensures that variables are bound when extrapolated in time. This formulation provides better accuracy and stability at a much larger time-step size compared to unbounded methods.

In ANSYS Fluent, the explicit formulation for volume fraction allows only first-order transient method, whereas implicit formulation allows both first and second order transient methods. For long-running simulations, the explicit formulation becomes impractical due to the time-step size constraint, whereas the implicit formulation along with the bounded second order transient method achieves better accuracy and stability at the larger time-step sizes.

2.4 Boundary Conditions

2.4.1 Upstream and Downstream: An open channel wave boundary condition is applied at a velocity inlet, which requires wave inputs depending on the type of wave. Typically, wave height and liquid depth are provided for modeling Solitary Wave, whereas wave height, wave frequency, phase difference and liquid depth are provided for modeling Stokes Wave. Averaged flow velocity is specified to model the effect of flow current. Hydrostatic pressure profile is applied at the pressure outlet boundary.

2.4.2 Numerical Beach: Numerical beach is defined in the region adjacent to the pressure outlet for suppressing the numerical reflections propagating upstream. The length of the beach zone can be provided in ANSYS Fluent with a start point and an end point in the specific directions. A damping source is added in the momentum equation. Damping effect gradually increases along the beach, whereas it gradually decreases away from the free surface. Damping source is given by,

$$S = -\left[C_1 \rho V + \frac{1}{2} C_2 \rho |V| V \right] f(z) f(x)$$
$$f(x) = r_x^2 \ , \quad f(z) = 1 - r_z$$
$$r_x = (x - x_s)/(x_e - x_s) \ , \quad r_z = (z - z_{fs})/(z_b - z_{fs})$$

Where, x is the distance along the flow direction, z is the distance from the free surface level, S and V are source term and velocity in the $\hat{z}$ direction, C_1 and C_2 are linear and quadratic damping resistances, $f(x)$ and $f(z)$ are damping terms. x_s and x_e are the start and end points of the damping zone in the $\hat{x}$ direction. z_{fs} and z_b are the free surface and bottom level along the $\hat{z}$ directions.

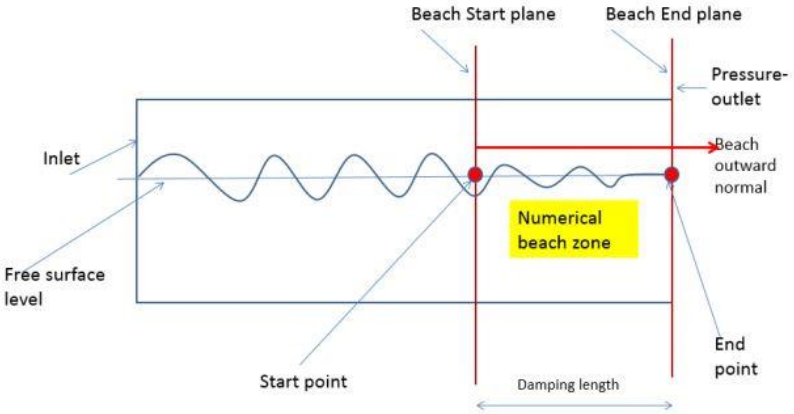


Figure 2: Schematic drawing of numerical beach zone [1]

3 Case Studies

3.1 Case study-1: Solitary Wave Propagation

The first objective of this case study is to demonstrate the sensitivity of 5th order Solitary wave at different relative heights. The second objective is to find best numerical methods to get accurate and mesh-independent results at a large time step size.

3.1.1 Problem Description

A two-dimensional rectangular domain of 40 m x 4 m is considered for this simulation. A numerical beach zone of 5 m from the outlet is used to avoid reflections. Four cases are studied with different wave heights as 0.4 m, 0.55 m, 0.6 m and 0.7 m respectively. Free stream velocity is taken as 0 m/s and liquid depth (d) is considered as 1 m for all the variants. Other important wave input parameters depending on different wave heights are tabulated below (see Table 1).

Maximum theoretical limit of relative wave height is 0.78 whereas, considering 5th order wave model, the maximum numerical limit is 0.55. All four cases were run with a time step size of 0.02 s and total simulation time is 7 s. Time step size equivalent to T/100 is very aggressive and close to T/200 provides reasonable results in the above cases. Total time is chosen based on the initialized waves crossing completely through outlet boundary.

Table 1

Scenarios	Wave Height, H (m)	Relative Height, (H/d)	Time-Period, T (s)
Scenario-1	0.4	0.4	3.8033
Scenario-2	0.55	0.55	3.2334
Scenario-3	0.6	0.6	3.0746
Scenario-4	0.7	0.7	2.7700

PISO scheme is used for P-V coupling, second-order upwind and compressive schemes are used for momentum and volume fraction respectively. First and Second order transient methods are used with Implicit formulation whereas only First order transient method is allowed with Explicit formulation.

3.1.2 Results and Discussion

A mesh independence study is carried out with four grid levels at a wave relative height of 0.4. Grid1, Grid2, Grid3, and Grid4 have 4K, 16K, 64K and 256K quad mesh elements respectively. Figure 3 shows that Grid1 and Grid2 results are deviating but Grid3 and Grid4 results are matched closely with analytical solution obtained from a 5^{th} order wave theory formulations, thus Grid3 is considered for further study.

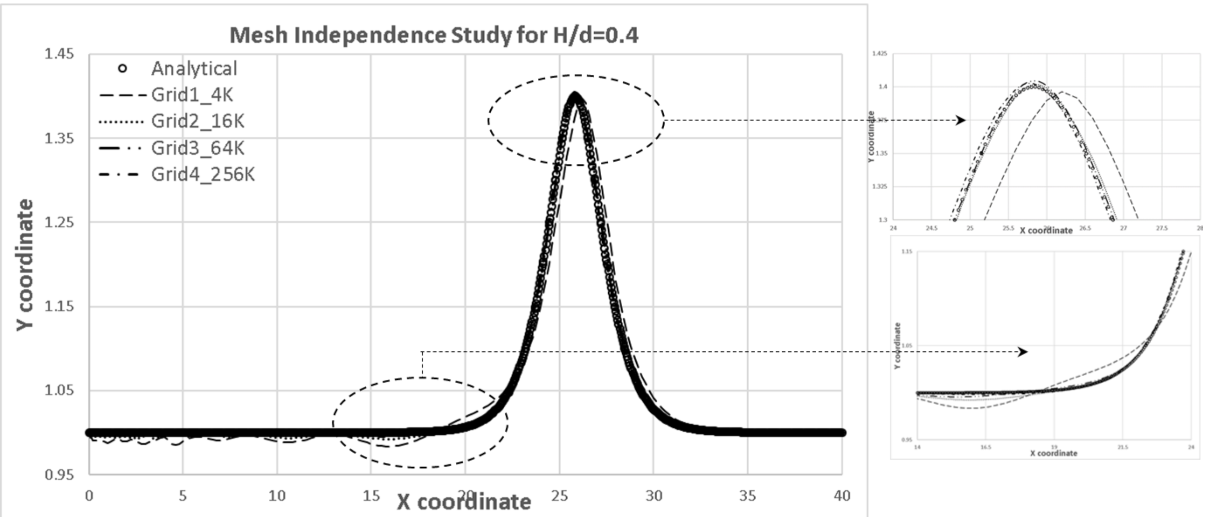


Figure 3: Mesh Independence study for H/d = 0.4

The volume fraction contour shows free surface level at 7 s for relative height of 0.4 (see Figure4).

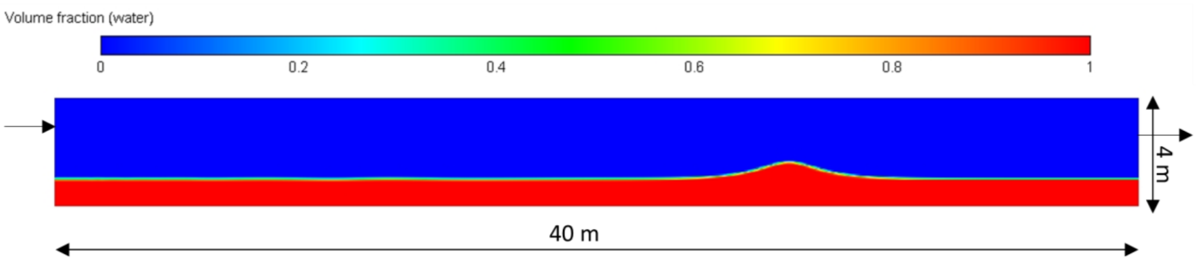


Figure 4: Free surface contours for wave propagation at relative wave height of 0.4 at t = 7s

Figure 5 shows the comparison of free surface profiles for different transient methods at the relative height of 0.4. Implicit volume fraction formulation with first-order transient method results in significant damping of the wave due to large numerical diffusion. Explicit formulation with first-order transient method matches well at the peak but deviates at tail side. Implicit formulation with second order time is closer to analytical results compared to other simulations. This study concludes that the Implicit formulation with second order time provides stable and accurate results in wave simulations.

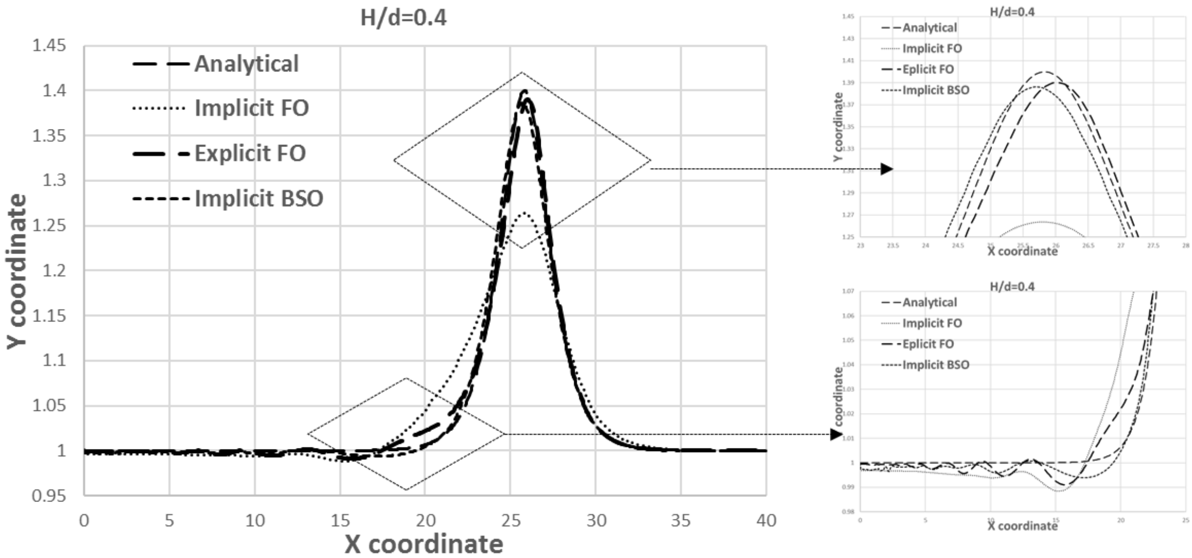


Figure 5: Comparison of free surface profiles for Implicit first order, Explicit first order and Implicit bounded second order formulations

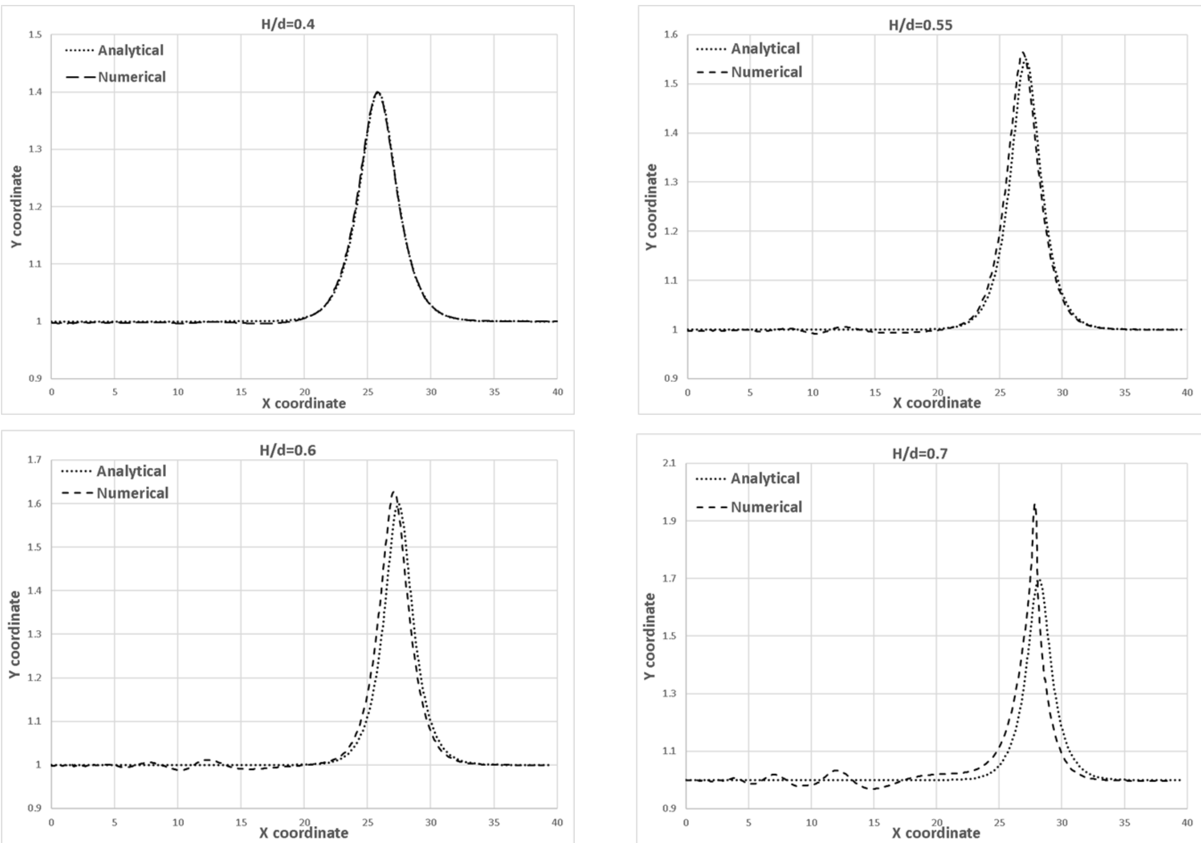


Figure 6: Free surface profile for solitary wave propagation at relative heights of 0.4, 0.55, 0.6 and 0.7

Figure 6 shows the comparison of free surface profiles with analytical solutions at different relative heights of 0.4, 0.55, 0.6 and 0.7. All cases are run on Grid3 and the Implicit formulation is used with second order time. Numerical results are in good agreement with analytical solutions up to a relative height of 0.55. Beyond this, wave profiles start showing unphysical large peaks and larger wiggles at the tail of solitary wave. This case results clearly indicate that wave relative height plays an important role and after a certain point, the numerical solution becomes unstable

3.2 Case study-2: High Steep Stokes Wave Propagation in Deep Sea

The objective of this case study is to demonstrate the sensitivity of time step sizes for Explicit and Implicit methods and the effectiveness of bounded second order time formulation to speed up the solution while maintaining high accuracy. The second part of this study is to show the sensitivity of highly steep Stokes wave using Skjelbreia and Fenton variants. In both variants, b_{ij} and c_i, the expressions of kH in Stokes wave theory are different. Though both the variants behave well for wave steepness below 0.1, at a high wave steepness and relative height, Fenton's variant is theoretically more robust, whereas the Skjelbreia wave variant results in predicting higher wave celerity at peak which further leads to wave breaking.

3.2.1 Problem Description

A two-dimensional rectangular domain of 1200 m x 130 m is considered for this case. Case is solved with two different grids having 211K and 844K quad mesh elements. Numerical beach length is set to 3 times of wavelength from the outlet to avoid reflections. The free stream velocity at the inlet is 2 m/s. Important wave input parameters related to this case are tabulated below (see Table 2).

Table 2

Wave Height, H	Wave Length, L	Wave Steepness, (H/L)	Time-Period, T (s)
14.4	120	0.12	7.1957

The maximum theoretical limit of wave steepness is 0.142. Using a 5^{th} order wave theory, a stable numerical limit of wave steepness is 0.1 and the maximum numerical limit is 0.12 which means wave steepness, in this case, is exceeding the stable numerical limit and waves could be stable or unstable in this regime.

3.2.2 Results and Discussion

Figure 7A compares the results for explicit formulation using first-order transient method at time step size of T/1000 and T/500, waves start breaking at T/500 and even show deviation with analytical at T/1000. It shows that explicit formulation is very sensitive at high wave steepness and requires a very small time-step size. Implicit formulation with first-order transient method is numerically diffusive and results in the decay of wave amplitudes (see Figure 7B). On the other hand, the Implicit formulation with bounded second order transient method predicts wave profiles with high accuracy at a much larger time-step size (T/254) (see Figure 7C). This study concludes that the Implicit formulation with bounded second order transient method is best suited for wave modeling to predict accurate results even at a high time step size.

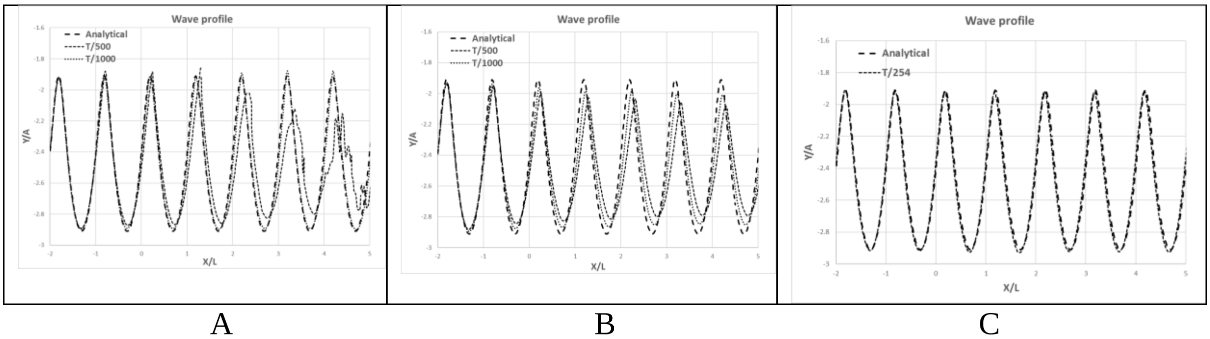


Figure 7: Wave Profiles with Explicit first order transient formulation (A), wave profiles with Implicit first order transient formulation (B) and wave profiles with Implicit bounded second order transient formulation (C)

Free surface contours and wave profiles obtained using Skjelbria and Fenton variants are compared and shown in Figure 8 and Figure 9 respectively. It is observed that wave profiles are almost same on coarse and fine mesh for both variants and grid refinement do not further improve the results. Wave free surface contours and profiles show breaking of waves in Skjelbria variant but no breaking of waves in Fenton variant. This case study concludes that the Fenton wave variant is advisable for high steep wave simulations.

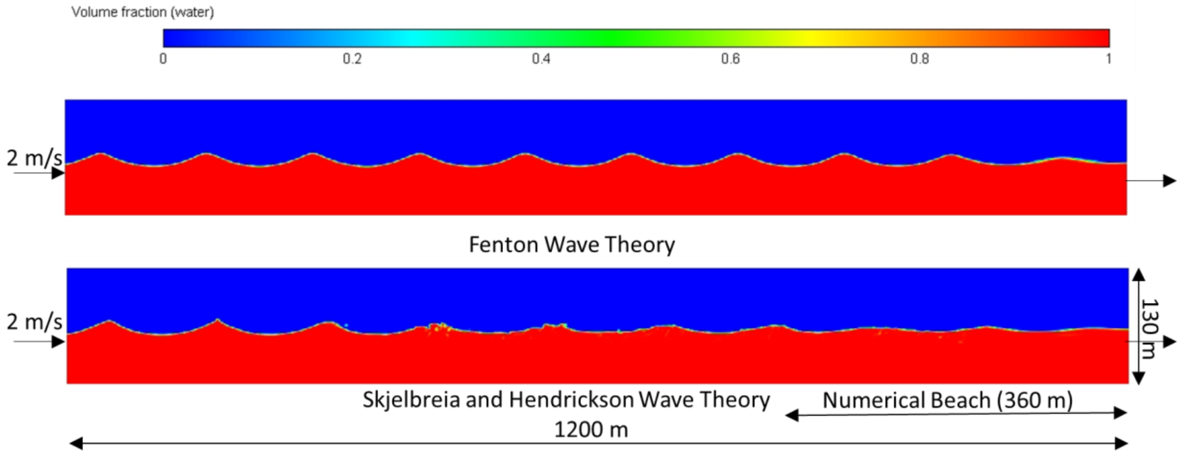


Figure 8: Free surface contours for wave propagation at shallow depth with Skjelbreia and Fenton variants.

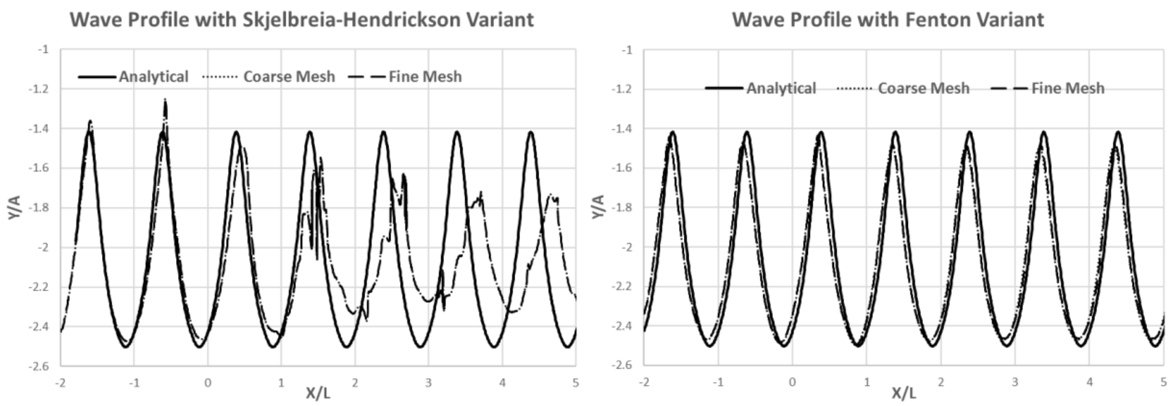


Figure 9: Comparison between Skjelbreia and Fenton variants for wave propagation at shallow depth.

3.3 Case study-3: High Steep Stokes Wave Propagation at Shallow Depth

The objective of this case study is to demonstrate the applicability of highly steep Stokes wave at shallow depth. This is an extreme scenario as Stokes wave is derived under the assumption of zero relative height therefore it is mostly applied to deep waves.

3.3.1 Problem Description

A two-dimensional rectangular domain of 30 m x 1.8 m is considered for this case. Case is solved on two grids having 540K and 2160K quad mesh elements respectively. Numerical beach is set as 10 m from the outlet to avoid reflections. Free stream velocity at the inlet is 2 m/s. This case is solved for 16 s with a time step size of 0.04 s which is very close to T/250, to achieve a stable solution. Important wave input parameters are tabulated (see Table 3). Fenton variant is considered because it provides a more accurate prediction than Skjelbreia variant as seen from case study 2.

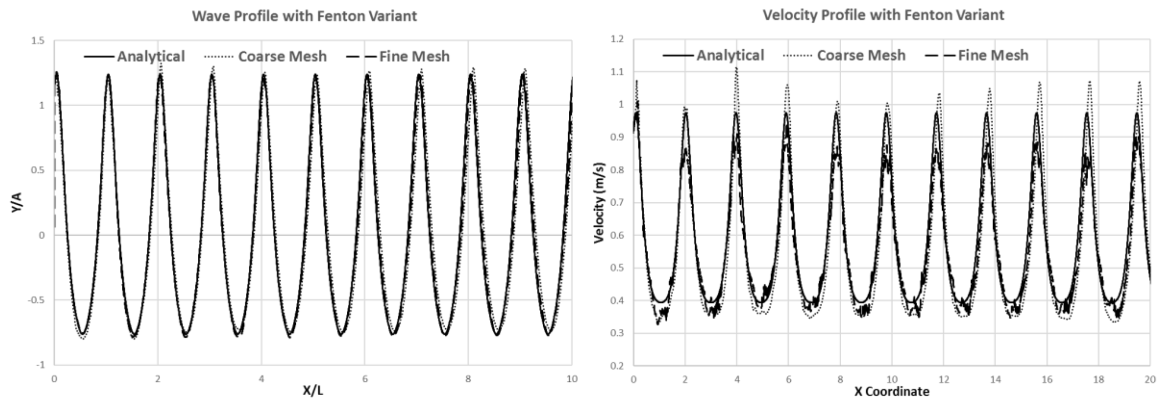
Table 3

Wave Height, H (m)	Wave Length, L (m)	Liquid Depth, d (m)	Relative Height, H/d	Steepness, H/L	Time-Period, T (s)
0.216	1.94	0.6	0.36	0.111	1.0635

In this case, the relative height and wave steepness are closer to the wave breaking limit, thus getting an accurate wave profile from the numerical simulation is very challenging. Implicit volume fraction formulation with second-order transient method is adopted in this case study because it maintains higher accuracy with larger time step size and robust at a high wave steepness as seen in case study 1.

3.3.2 Results and Discussion

Though both wave steepness and relative height are higher in this case, wave profile shows a good match with analytical (see Figure 10A). Wave profiles are consistent on coarse and fine mesh. Velocity profiles of water are compared with analytical (see Figure 10B). VOF method solves the velocity for both water and air phases at the free surface. Due to a lighter density, air velocity is higher at the free surface compared to water. Numerical instability is seen in velocity profiles of water near the free surface as the presence of air is felt in water velocity profiles near the peak.



A B

Figure 10: Numerical and analytical comparison for wave profile (A) and velocity profile (B)

This case study concludes that the Fenton wave variant adopted in ANSYS Fluent can be used up to a relative a height of 0.36 with a high wave steepness of 0.11 even the wave theory derivation ignores the effect of relative height. Implicit formulation along with bounded second order transient method is crucial to get accurate results.

4 Conclusions

The choice of an appropriate wave theory along with its order is the primary criterion for modeling waves within stability limits. 5^{th} order solitary wave theory can be used for relative heights of 0.55-0.6 as compared to the theoretical limit of 0.78. 5^{th} order Stokes wave theory can be used for wave steepness of 0.12 as compared to the theoretical limit of 0.142. In the present study using ANSYS Fluent VOF method and wave models, it has been shown that the Stokes wave theory can be applied in the shallow regime with a relative height of 0.36 and wave steepness of 0.12 without wave breaking. Different variants of wave theories could perform differently near breaking conditions and it has been shown that Fenton's variant of Stokes wave is more stable than Skjelbreia variant for high steep waves.

On the other hand, the choice of appropriate free surface numerics also plays a vital role in predicting accurate wave behavior, especially near wave breaking conditions. It has been concluded that implicit formulation of volume fraction along with second order transient method implemented in ANSYS Fluent is very accurate and robust in carrying out simulations with a large time step size. The Explicit formulation is very sensitive near breaking conditions and becomes impractical due to the requirement of very small time-step size. Thus, this paper consolidates the best numerical settings and wave models to achieve better prediction of wave profiles near wave breaking conditions for high steep waves.

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