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Numerical Methodology for the Prediction of Supersonic Jet Noise

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Abstract: The Mach wave radiation give rise by the instability waves in a supersonic axisymmetric jet with Ma=2.1 is investigated. Near field of supersonic axisymmetric jet is computed by nonlinear disturbance equations ,while sound radiation in far field is computed by Wu's Extrapolation of Integral. The results are in good agreement with experiment data and DNS. Present study can analysis the Mach wave radiation both quantitative and qualitative.

Keywords: Mach wave radiation, instability wave, supersonic jet flow, Wu's extrapolation of integral.

1 Introduction

Jet noise remains a major source of aircraft noise. But there are few engineering tools to predict supersonic jet noise rapidly and cheaply, partly because the detailed mechanisms are not well understood. Recent theoretical investigations have described that instability waves, or in a broader sense large-scale orderly structures, constitute a dominant source of jet noise. In the supersonic jets, the role of supersonic modes in radiating Mach waves was confirmed by a series of experiments[11].
Tam and Burton[10] applied the linear stability theory on relates the far-field sound to the growth and decay of instability waves in the jet flow. In predicting supersonic mixing noise, two prevalent approaches are Lighthill's equation and the Kirchhoff surface method. Base on asymptotic theory, Wu[12] concluded that radiated Mach waves in the far field is determined explicitly in terms of amplitude function.
Our goal is to give the quantitative description of the radiated sound field, its prediction and its connection to sound-generating structures in the supersonic jet flow. In this effort, we used nonlinear disturbance equations to simulate the instability waves of a supersonic jet and to analyze the associated Mach wave radiation by using Wu's amplitude equations. This study aims at underlying mechanisms contained in the growth and decay of instability waves that give rise to intense Mach wave radiation quality as well as quantity.

2 Numerical Methodology

2.1 Governing Equation

We consider a supersonic axisymmetric jet flow, The governing equation derived from compressible Navier-Stokes equations for the conservative variables is given below:

$$\frac{\partial U'}{\partial t}+\frac{\partial E'}{\partial x}+\frac{\partial F'}{\partial r}=\frac{\partial E_v'}{\partial x}+\frac{\partial F_v'}{\partial r} \qquad (2.1)$$

where $U'=\begin{pmatrix} r\rho' & r(\rho u)' & r(\rho v)' & re' \end{pmatrix}$

$$E'=\begin{bmatrix} r(\rho u)' \\ r\left[\rho_0 u_0 u' + (\rho u)'(u_0+u') + p'\right] \\ r\left[\rho_0 u_0 v' + (\rho u)'(v_0+v')\right] \\ r\left[h_0 v' + h'(u_0+u')\right] \end{bmatrix}, \qquad F'=\begin{bmatrix} r(\rho v)' \\ r\left[\rho_0 v_0 u' + (\rho v)'(u_0+u') + p'\right] \\ r\left[\rho_0 v_0 v' + (\rho v)'(v_0+v')\right] \\ r\left[h_0 v' + h'(v_0+v')\right] \end{bmatrix}$$

$$E_v'=\begin{bmatrix} 0 \\ r\tau_{xx}' \\ r\tau_{xr}' \\ r\left[(u\tau_{xx})' + (v\tau_{xr})' + q_x'\right] \end{bmatrix}, \qquad F_v'=\begin{bmatrix} 0 \\ r\tau_{rx}' \\ r\tau_{rr}' \\ r\left[(u\tau_{rx})' + (v\tau_{rr})' + q_r'\right] \end{bmatrix}$$

The convection terms are evaluated by a third-order upwind scheme, and sixth-order-accurate central scheme is used for diffusive terms. Time marching is achieved by a third-order Runge-Kutta scheme. The code has previously been used in our group and has been validated by some published paper.

2.2 Extrapolation of integral for the acoustic field

Thus far only the near field (hydrodynamic field) has been studied. Since we focus on predicting the far-field sound, it is important to describe a method of computing the sound radiation from the supersonic instability modes at this stage. An extrapolation of integral proposed by Wu (2005) was introduced. In the theoretical study, Wu(2005) focused on Mach wave radiation of nonlinearly evolving instability modes in supersonic jet. According to his model, the Mach wave field consists of two region: a near field and a far field, which is illustrated schematically in figure 1. (X. Wu, 2005)

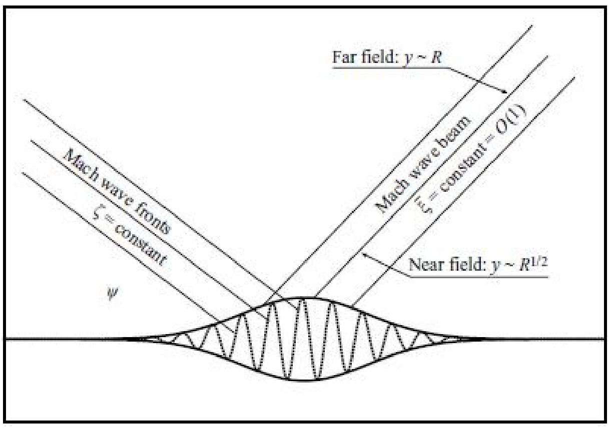


Figure 1: A sketch illustrating the radiation process and the structure of the Mach wave field.

And then, the perturbation is determined explicitly in terms of amplitude A.

$$\overline{p}_0 = \frac{\ell_\infty}{\sqrt{\overline{r}}} A(\overline{\xi}, \overline{\eta}) \qquad (2.2)$$

with

$$\overline{\xi} = \overline{x} - \overline{r}/q, \quad \overline{\eta} = \overline{t} - \left(M_a^2 c/q\right)\overline{r} \qquad (2.3)$$

Thus, a Mach wave is formed as the phase of the supersonic modes propagating along the characteristic $\overline{\eta} = coms \tan t$, while its envelope (acoustic energy) propagates along the characteristic $\overline{\xi} = coms \tan t$.

Extended to far field, an appropriate solution of the radiated Mach wave is found to be:

For a single wave,

$$\widetilde{p}_0(\overline{\xi}, \widetilde{r}) = \frac{e^{-\pi i/4}}{\widetilde{r}} \left(\frac{\alpha q^3}{2\pi M_a^2 c^2} \right)^{1/2} \ell_\infty \int_{-\infty}^{\infty} A(\zeta) \exp\left\{ \frac{\alpha q^3}{2 M_a^2 c^2 \widetilde{r}} (\overline{\xi} - \zeta)^2 \right\} d\zeta \qquad (2.4)$$

For the wavetrain,

$$\hat{p}_0(\overline{\xi}, \widetilde{\omega}, \widetilde{r}) = \frac{e^{-\pi i/4}}{\widetilde{r}} \left(\frac{\alpha q^3}{2\pi M_a^2 c^2} \right)^{1/2} \ell_\infty \hat{Q}(\overline{\xi}, \widetilde{\omega}, \widetilde{r}) \exp\left\{ -\frac{i M_a^2 \widetilde{\omega}^2 \widetilde{r}}{2 \alpha q^3} \right\} \int_{-\infty}^{\infty} A(\zeta) \exp\left\{ \frac{\alpha q^3}{2 M_a^2 c^2 \widetilde{r}} (\overline{\xi} - \zeta)^2 \right\} d\zeta$$

$$\hat{Q}(\overline{\xi}, \widetilde{\omega}, \widetilde{r}) = \int_{-\infty}^{\infty} \hat{A}(\zeta, \ \widetilde{\omega}) \exp\left\{ \frac{i \alpha q^3}{2 M_a^2 c^2 \widetilde{r}} \left(\overline{\xi} - \zeta + \frac{M_a^2 c^2 \widetilde{\omega} \widetilde{r}}{\alpha q^3} \right)^2 \right\} d\zeta \qquad (2.5)$$

So far, the Mach wave field would be fully determined if amplitude function *A* is known. The idea of matching the far acoustic field with the near hydrodynamics structures is a fundamental principle in aeroacoustic approach. With above analyses, the process of Mach wave radiation by supersonic modes are clear. A feasible method for predicting the acoustic field is:

In the near field, the evolving of supersonic modes could be gained numerically in §2.1. Meanwhile, formula (2.4) and (2.5) are the extrapolation of integral for the acoustic far-field. The "inner" and "outer" region match by characteristics (2.3).

2.3 Mean flow and supersonic modes

In our numerical computation, an initially supersonic axisymmetric jet flow with a velocity profile has been chosen to match the experimental data of Tam & Burton (1984a, b).

$$U = \begin{cases} 1 & r \le h(x) \\ \exp\left\{ -\left(\dfrac{r - h(x)}{b(x)} \right)^2 \right\} & r > h(x) \end{cases} \qquad (2.6)$$

For simplicity the Prandtl number is assumed to be unity so that the temperature profile is given by Crocco's relation. We defined the Reynolds number, Mach number and Strouhal number as:

$$\mathrm{Re} = \rho_J U_J R_J / \mu_J, \quad Ma = U_J / c_J, \quad St = 2 R_J f^* / U_J$$

Where c_J is the sound speed at the jet exit.

Disturbances are taken in the form of normal modes at the inflow boundary

$$\phi(x, r, t) = \hat{\phi}(r) \exp[i(\alpha x - \omega t)] + c.c \qquad (2.7)$$

where $\phi = (\rho' \quad u' \quad v' \quad T')$. $\hat{\phi}(r)$ is an eigenfunction (shape of the mode), ω is a real angular frequency. Eigenfunctions and wavenumbers are obtained from spatial linear stability analysis for a particular frequency and mode number. Disturbances are added to all flow variables at the inflow boundary.

3 Results

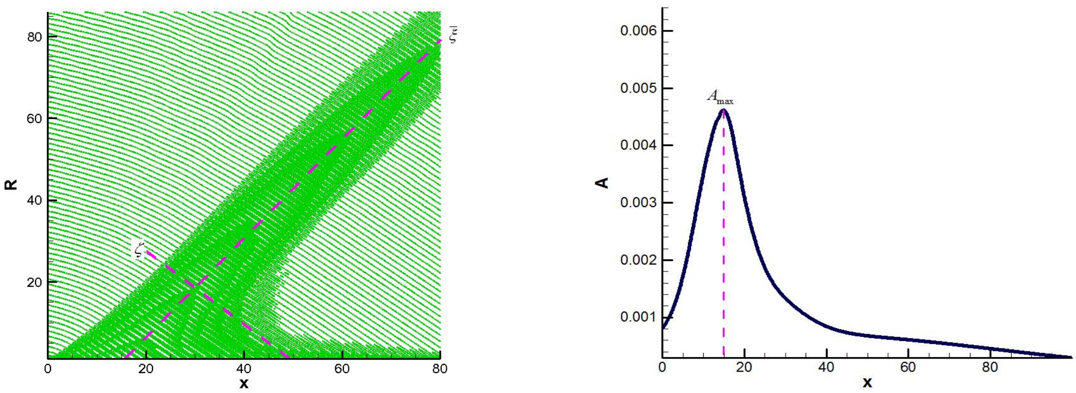
3.1 A single mode

In this section the far-acoustic field generated by a single wave is studied in detail. Calculations were carried out for different amplitudes and frequencies of the inflow disturbances listed in Table 1. Extrapolation boundary condition was used for outflow. Only the evolution of axisymmetric perturbations *(m = 0)* were computed, and symmetry conditions on the jet axis were imposed accordingly. Simulation were performed for *Re=7000* and *Ma=2.1*.

Table 1 The inflow instability wave characteristics

St	ω_0	α_r	$-\alpha_i$	A_0
0.4	1.2560	1.6478	0.4985	0.0001
0.5	1.5707	2.3051	0.5759	0.0001
0.6	1.8850	2.841	0.610	0.0001

We start with an examination of the Mach wave radiation by an instability wave with *St=0.4* . The goal of the first part of the results is to verify the accuracy of the current approach. A notable feature can be seen in Figure 1(a): the Mach wave propagated along the characteristics ζ, while its envelope (acoustic energy flux) propagated along the characteristics ξ. Therefore the Mach wave beam was perpendicular to the Mach wave front. The origin of sound emission can be traced back to the evolution of the instability mode, as shown in Figure1(b). The disturbance undergo the growth and reach its saturation, then decay. The Mach wave emitted as if emanating from the streamwise location where the amplitude of the instability mode was maximum.

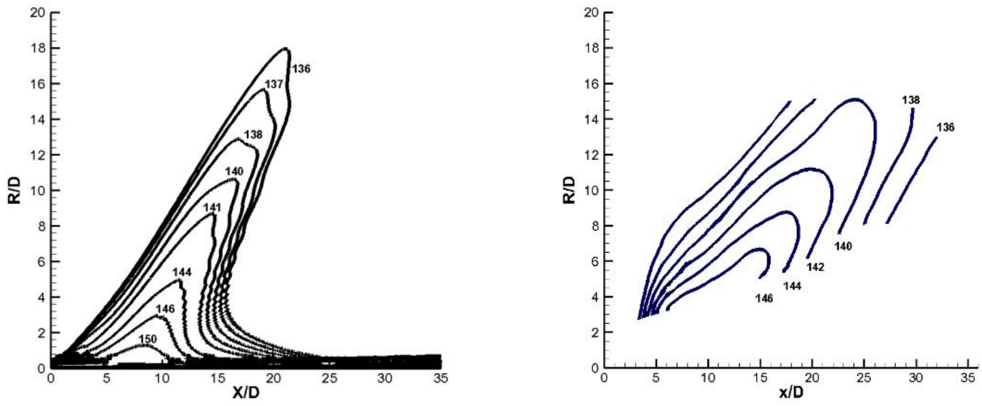


(a) (b)

Figure 2: (a) Pressure contours of Mach waves in far field; (b) Amplitude of pressure fluctuation in near field.

$$\psi = \sin^{-1} \frac{1}{M_a c} \qquad (3.1)$$

Thus the Mach wave fronts are parallel lines with the angle of upstream axial direction, as shown in formula(3.1). This result is consistent to the observed in the experiments of Troutt & Mclaughlin, $\psi \approx 56^0$ for *St=0.4* , as well as the theory. For further comparison, the results of sound pressure level (SPL) was calculated and found in exact agreement with experiment data (Figure 3a, b).

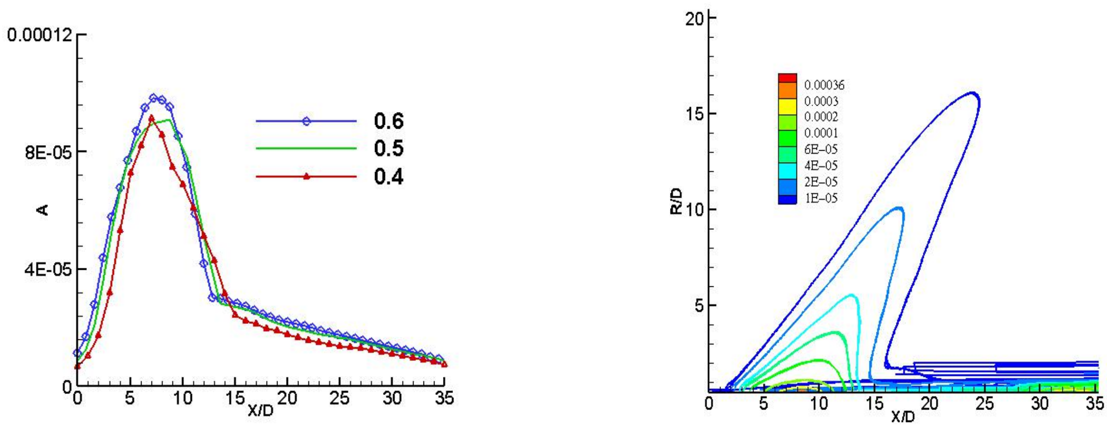


(a) (b)

Figure 3: SPL calculated by current approach (a), at *St=0.4;* (b) Experiments of Troutt & Mclaughlin.

3.2 Wavetrain

In this section we investigate the modulation wavetrain. First case is an linear wavetrain. The instability waves, for *St=0.4,0.5,0.6,* have a very small amplitude, so a Gaussian envelop is assumed at inlet. The contours of pressure are shown in Figure 4(b). By comparing with Figure 2(a), A close resemblance is evident. In present case, the Mach waves are stronger and Mach wave beam are slightly broader, and also reveals clear direction.



(a) (b)

Figure 4: (a) Amplitude of pressure fluctuation in near field；(b) Pressure contours of Mach waves in far field.

4 Conclusion and Future Work

It can be note that the present analysis and approach can successfully predict the quantitative as well as qualitative features of the Mach wave radiation associated with supersonic instability waves. It must be pointed out that the analysis for the Mach waves are also valid for an instability wave packet. A more complete analysis of the wave packet will be presented in the future work.

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