Tenth International Conference on Computational Fluid Dynamics (ICCFD10), Barcelona, Spain, July 9-13, 2018

ICCFD10-2018-xxxx

A Study on the Performance of Implicit Time Integration for the Navier-Stokes Equations using the Discontinuous Galerkin Spectral Element Method

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1 Introduction

An implicit time integration is presented for solving the unsteady compressible Navier-Stokes equations spatially discretized by a high-order Discontinuous Galerkin Spectral Element Method (DGSEM). In particular, we consider implicit Runge-Kutta methods as Explicit step Singly Diagonally Implicit Runge-Kutta (ESDIRK) schemes and a fixed preconditioned matrix-free Newton-Krylov solver for the appearing non-linear systems. We investigate and compare the efficiency of different preconditioners, which are based on the analytically computed block Jacobi, exploiting the tensor product structure of DGSEM. For this study we also consider the effects of CFL number, polynomial degree, space dimension and Reynolds number. To evaluate the performance we compare the computing time of the implicit scheme to an explicit Runge-Kutta time integration. Depending on the setup and the choice of parameters we show that the computational efficiency of the implicit solver can be increased and outperform the explicit solver in terms of CPU time.

2 Numerical Approach

In the case of the Navier-Stokes equations the method of lines approach leads us to non-linear equation systems of the form

$$u_h - \Delta t R(u_h) = r, \tag{1}$$

which arise by the implicit treatment of the time integration. R describes the spatial operator resulting from the Discontinuous Galerkin method. We remark that the operator R is already divided by the mass matrix, which is diagonal for DGSEM due to the collocation property. For ESDIRK schemes we obtain a sequence of non-linear systems (1) according to the number of stages. Applying Newton's method, the implicit method requires the solution of linear systems of the form

$$(I - \Delta t J)x = b \tag{2}$$

in every Newton iteration, where J is the Jacobian matrix of the spatial operator R. This is performed by a restarted GMRES algorithm in a matrix-free context. For increasing the efficiency, the linear systems are preconditioned by the block Jacobi containing only the block diagonal elements of the Jacobian J. This means that there are no influences of the neighbouring elements included. However, this element-wise block structure reduces the computational cost for assembling the preconditioner as well as the required memory. For the element-wise solution of the preconditioned system, we compare different methods such as the LU decomposition and the ILU(0) factorization of the diagonal blocks. Since we consider the linear form (2), we have already included implicitly a so called mass-matrix preconditioner as used explicitly in [1]. Our computation of the block Jacobi is based on the tensor product structure within the DGSEM, implemented in the open source PDE framework Flexi[1]. Hence, we use the Kronecker products of small one dimensional matrices for efficient forming the diagonal blocks of the block Jacobi as in [2].

Good approximations of the Jacobian can strongly reduce the number of GMRES iterations, but are costly in their application. The big challenge with implicit solvers is to find a competitive trade-off between the approximation quality of the Jacobian and the efficiency of the application.

Numerical results are shown for the compressible Navier-Stokes equations in 2D and 3D, such as a travelling vortex in laminar flow and vortex shedding behind a cylinder. The GMRES iterations as well as the runtime performance are compared between the different preconditioners for multiple settings. Also the building and application time of the preconditioners will be taken into account. We conclude with showing the competitive runtime performance of an implicit solver in contrast to an explicit solver.

References

[1] Laslo T. Diosady and Scott M. Murman. Tensor-product preconditioners for higher-order space-time discontinuous Galerkin methods. *Journal of Computational Physics*, 330:296 – 318, 2017.

[2] Will Pazner and Per-Olof Persson. Approximate tensor-product preconditioners for very high order discontinuous Galerkin methods. *Journal of Computational Physics*, 354:344–369, 2018.

[1] Homepage: www.flexi-project.org, Github: github.com/flexi-framework/flexi