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Oral presentation | Reduced order models

## Reduced order models-I

Mon. Jul 15, 2024 10:45 AM - 12:45 PM Room C

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### [1-C-02] Large-eddy Simulations and Reduced-Order Modeling for NACA4412 Flow Near The Stall Angle

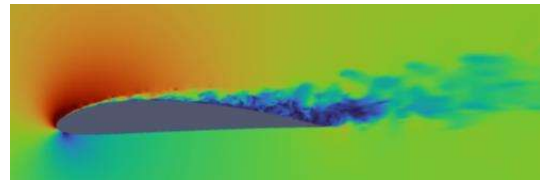
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Keywords: LES, DMD, ROM, POD

# Large-eddy Simulations and Reduced-Order Modeling for NACA4412 Flow Near The Stall Angle

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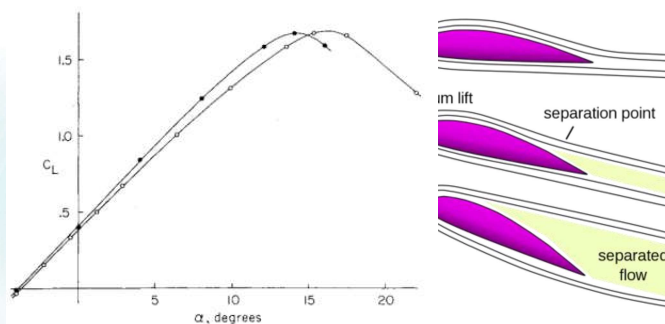
ICCFD12, Kobe, Japan  
July 15, 2024



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**Motivation:** Wings at high angle of attack

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# Flow Features

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- Vortical Structures
- Separation
- Instability
- Transition
  - K-H Rolls
  - Lambda Vortices

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# Large-eddy Simulation

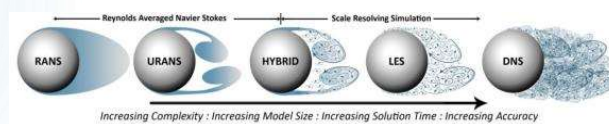
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Low-order methods (RANS) cannot resolve unsteady dynamics accurately.

LES is a suitable approach to capture unsteady shock-turbulence interactions.

Simulation of a high  $\alpha$  wing flow at high  $Re$  is challenging using LES due to several scales involved.

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Hart  
(2016)

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# NACA 4412

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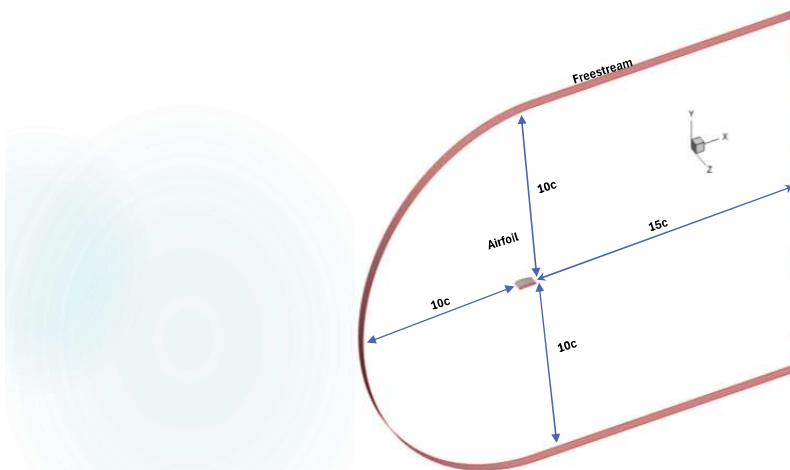


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# Computational Domain

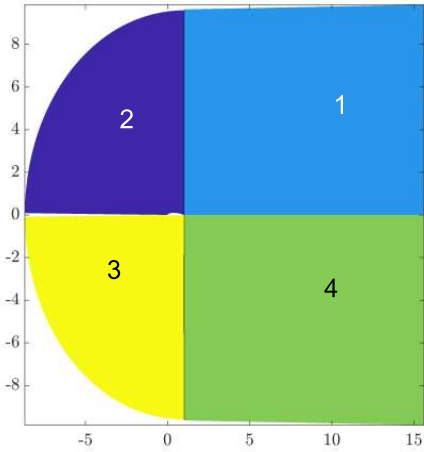
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# Computational Mesh

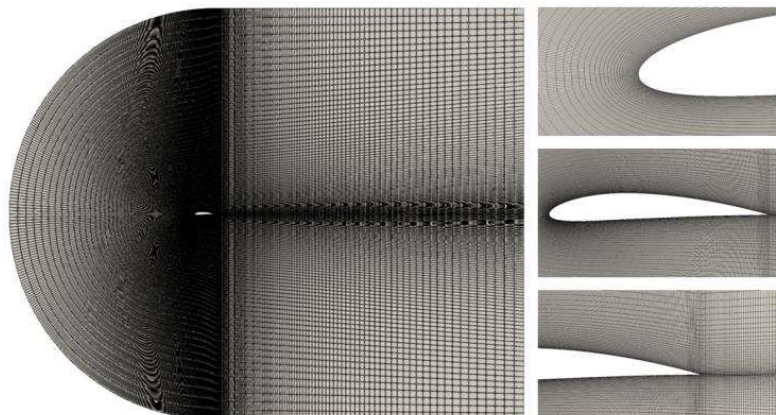


Block	Nx	Ny	Total
1	555	187	103785
2	121	187	22627
3	121	187	22627
4	555	187	103785
Total			252824

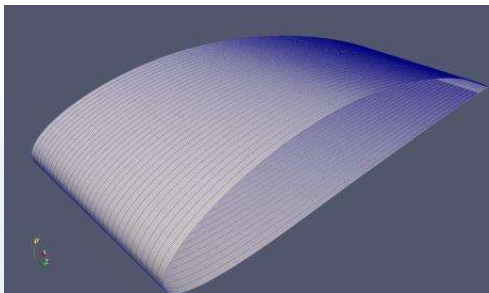
Block-structured (C-grid) Mesh

No. of Cells (Coarse) = 0.25 M  
No. of Cells (Fine) ~ 1 M

# Computational Mesh



# Computational Mesh (3D)



Spanwise BC = Symmetry

Block	Nx	Ny	Nz
1	555	187	32
2	121	187	32
3	121	187	32
4	555	187	32
Total			8090368

Block-structured (C-grid) Mesh

No. of Cells (Coarse) = 8M  
No. of Cells (Fine) ~ 24 M (Nz=96)

# Solver Setup

- ❑ RANS (2D)
  - ❑ simpleFOAM
  - ❑ Steady-state solver
  - ❑  $k - \omega$  SST model
  - ❑ Time scheme: First Order (Euler)
- ❑ URANS (2D)
  - ❑ rhoPimpleFOAM
  - ❑ Transient solver
  - ❑  $k - \omega$  SST model
  - ❑ Time scheme: First Order
- ❑ LES (3D)
  - ❑  $k - \omega$  equation
  - ❑ rhoPimpleFOAM
  - ❑  $k$ -equation model
  - ❑ Time scheme: Second-order (Backward)

All simulations performed using OpenFOAM-v2212

# Dynamical $k$ -equation Model

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$$\frac{\partial k_{SGS}}{\partial t} + \frac{\partial u_i k_{SGS}}{\partial x_i} = 2\nu_{SGS} |\bar{D}_{ij}|^2 - C_e \frac{k_{SGS}^{3/2}}{\Delta} + \frac{\partial}{\partial x_i} \left( \nu_{SGS} \frac{\partial k_{SGS}}{\partial x_i} \right) + \nu \frac{\partial^2 k_{SGS}}{\partial x_i \partial x_i}$$

$\nu_{SGS} = C_k \Delta \sqrt{k_{SGS}}$  represents the SGS kinematic viscosity,  
 $C_e = 1.048, C_k = 0.094$  are model constants,  
 $\bar{D}_{ij}$  denotes the filtered strain rate tensor.

- Instead of assuming local equilibrium (production=dissipation), this model solves a transport equation for  $k$
- Improves performance in complex flow situations with non-equilibrium turbulence by accounting for the effects of production, dissipation, and diffusion over time
- Unconditionally stable also in complex geometries without any homogeneous directions

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## Flow parameters

- $Re = 1.5 \times 10^6$
- $Ma = 0.07$
- $\alpha = 14^\circ$

## Boundary Conditions

- Velocity
  1. Freestream – freestreamVelocity
  2. Airfoil – No Slip
- Pressure
  1. Freestream – freestreamPressure
  2. Airfoil – ZeroGradient

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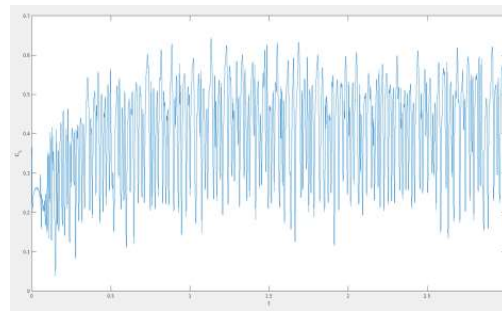
## Validation at low angle of attack

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- Angle of Attack =  $0^\circ$
- Solver = rhoPimpleFoam
- $\Delta t = 2 * 10^{-5}$  seconds

$C_L$ (Exptl)	$C_L$ (Simulation)
0.415	0.404

23 Coles, Donald, and Alan J. Woodcock, "Hydrodynamic study of flow past an NACA 4412 airfoil at maximum lift," AIAA Journal 17.6 (1979): 321-329.

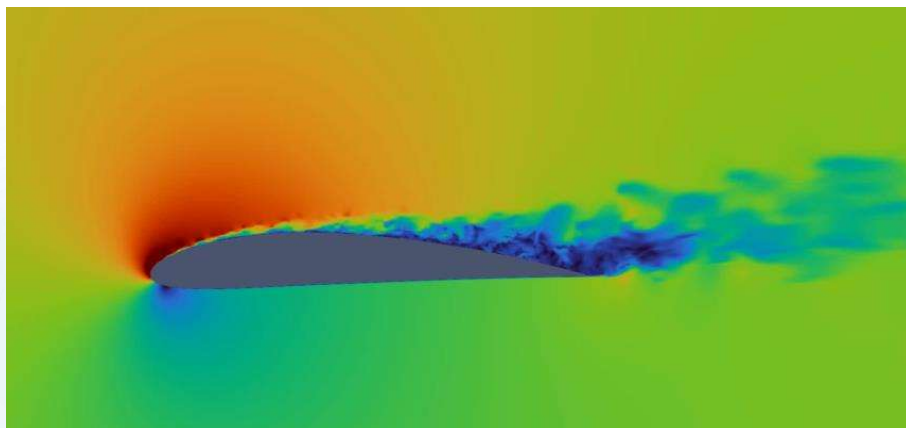


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## LES predictions

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# LES predictions

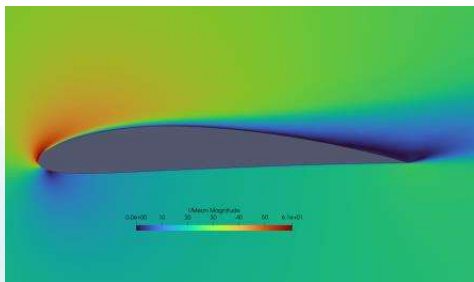


Figure.  $U_{mean}$  flood contour around the airfoil

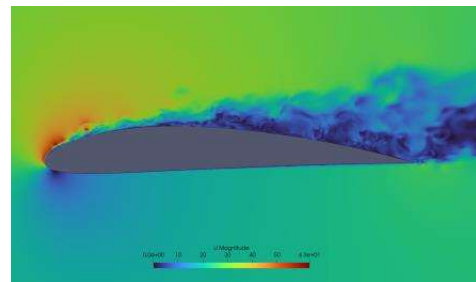
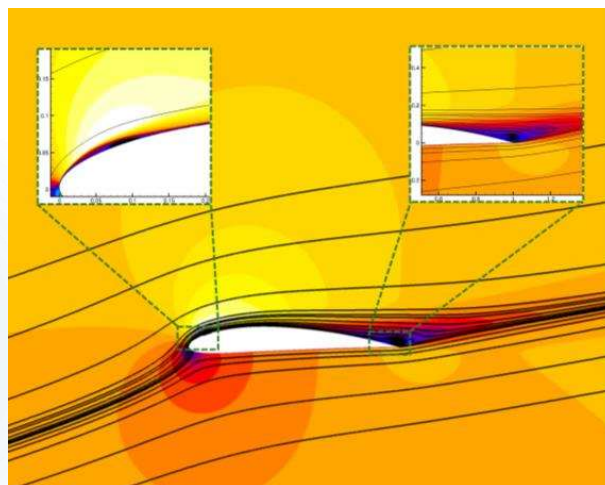


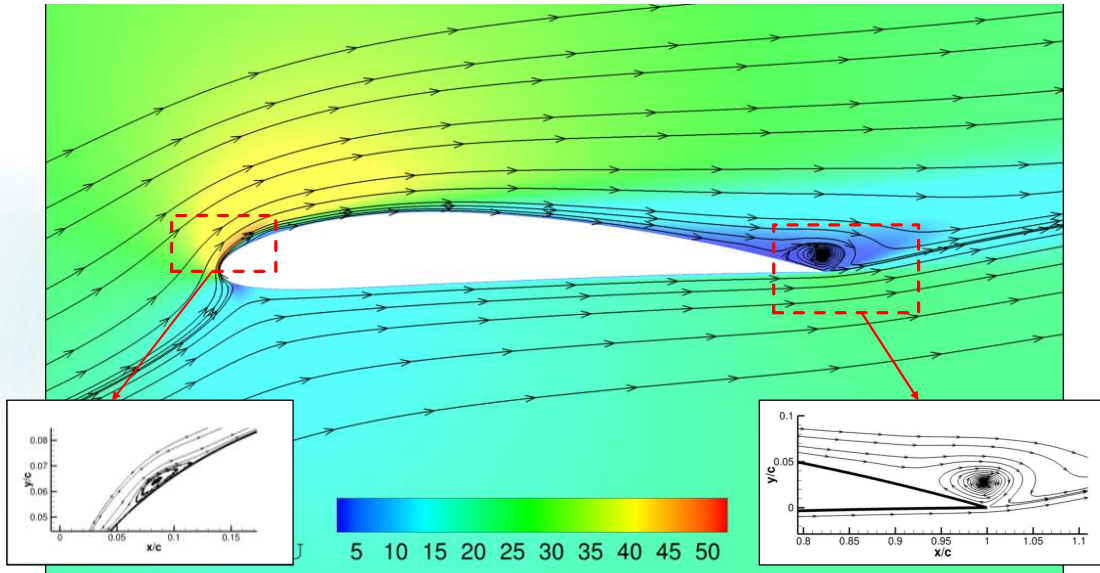
Figure. Flood contour showing  $U_{instantaneous}$

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# Mean Flow (LES)



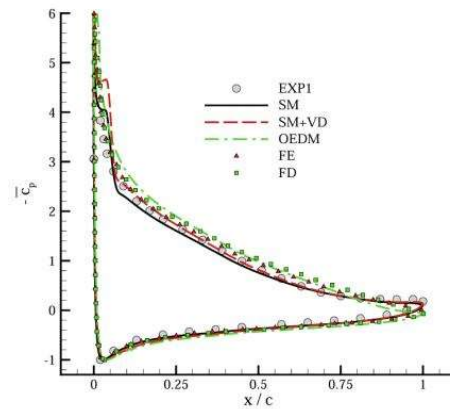
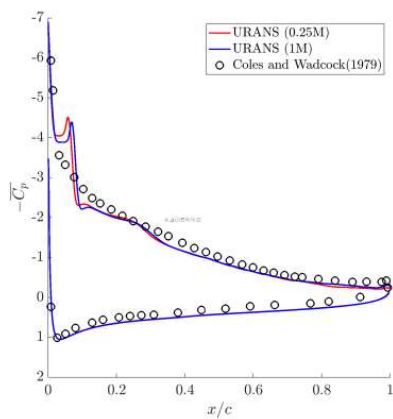
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## Grid Convergence

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Schmidt, Franke & Thiele (2001) -  $\alpha = 12^\circ$

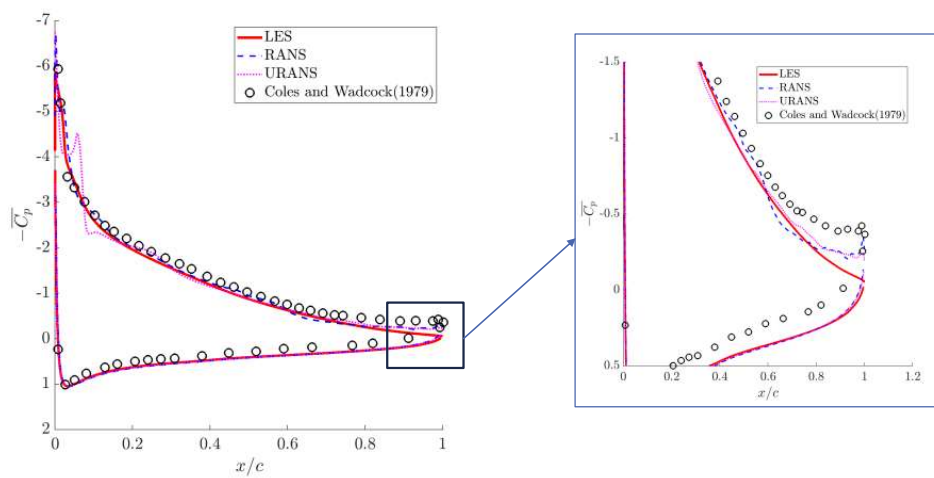
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## Mean Flow Features

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## Pressure Prediction

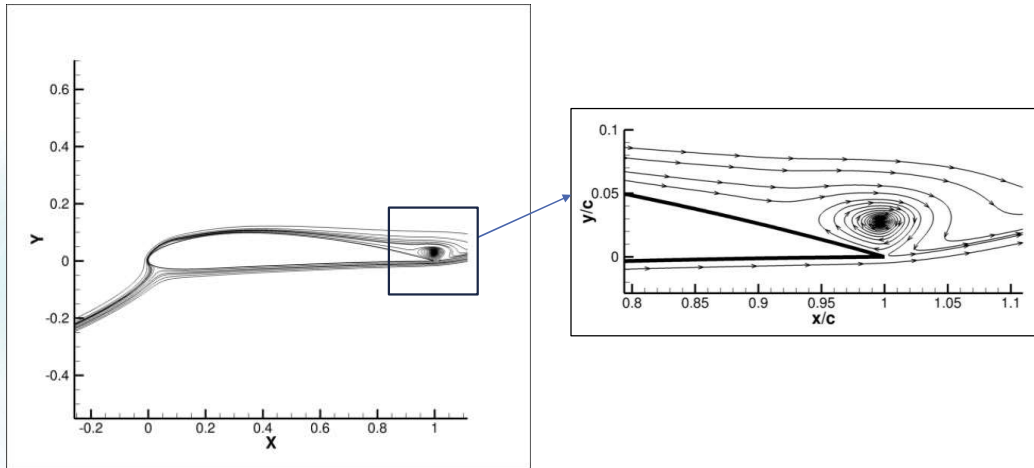


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# LES

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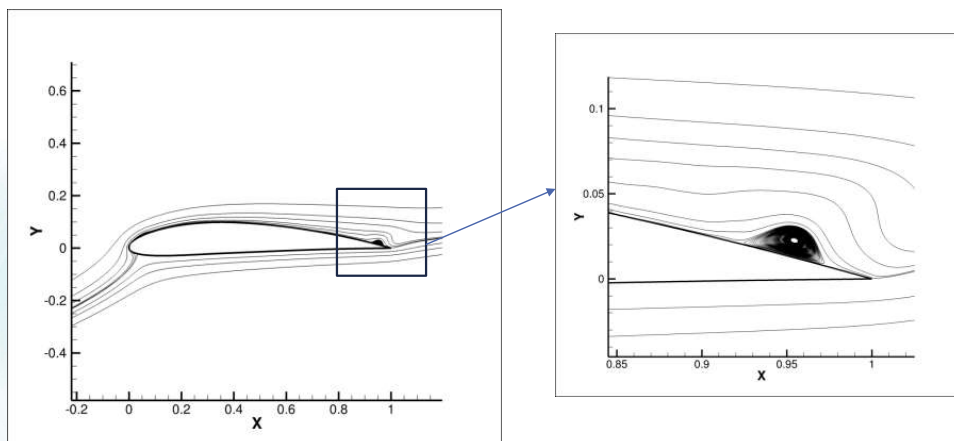


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# URANS

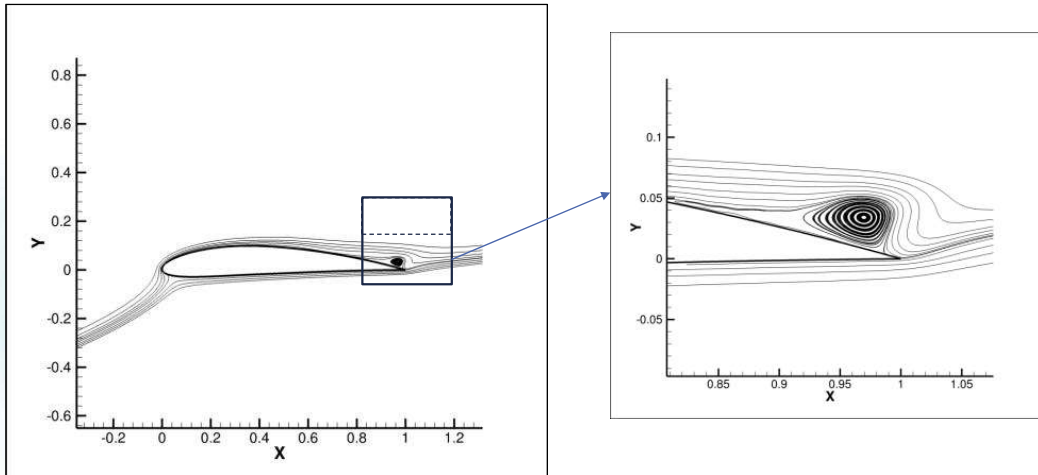
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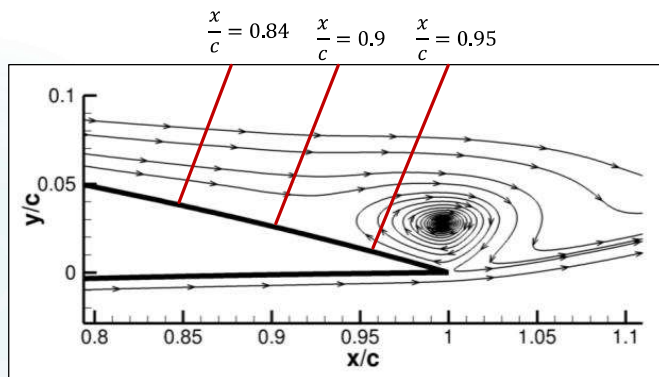
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# RANS



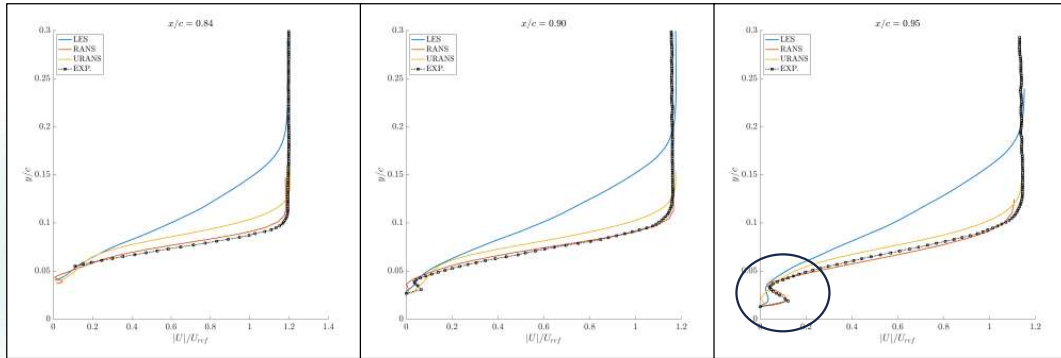
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# Velocity Profiles



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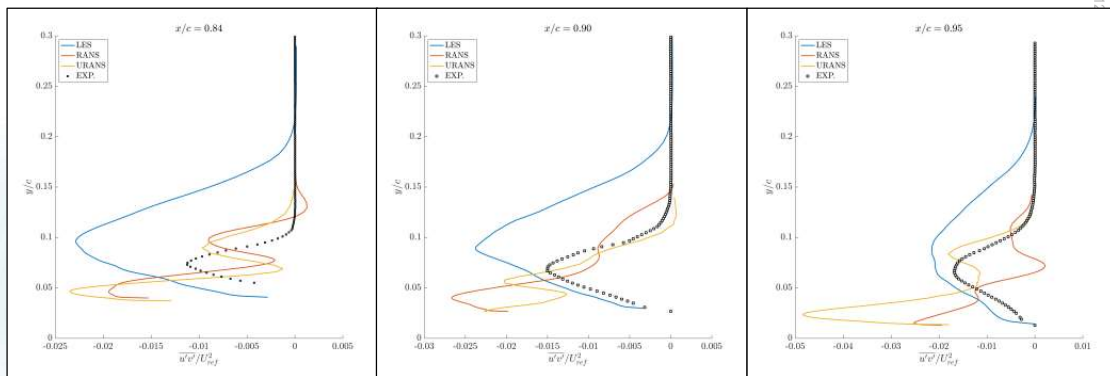
# Velocity Profiles



Experimentally measured separation point at  $\frac{x}{c} \approx 0.86$

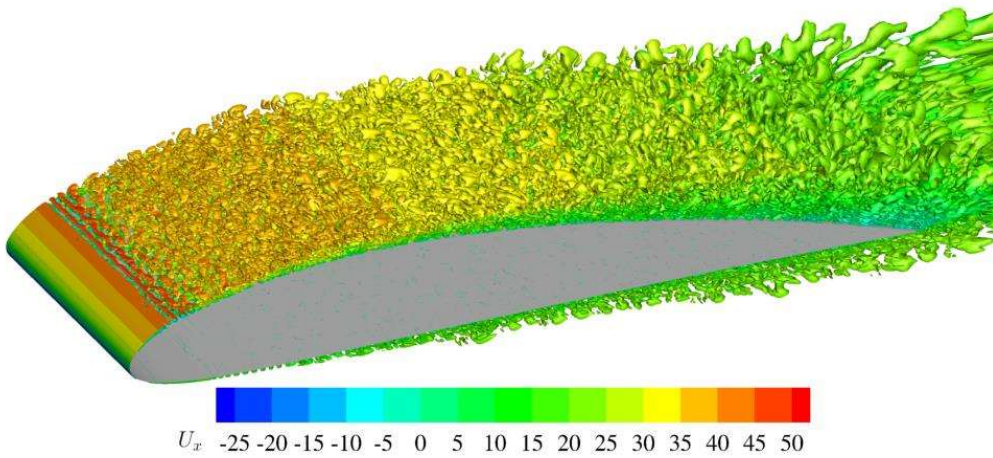
Exp.: Coles, Donald, and Alan J. Wadcock. "Flying-hot-wire study of flow past an NACA 4412 airfoil at maximum lift." *AIAA Journal* 17.4 (1979): 321-329.

# Reynolds Stress Comparisons

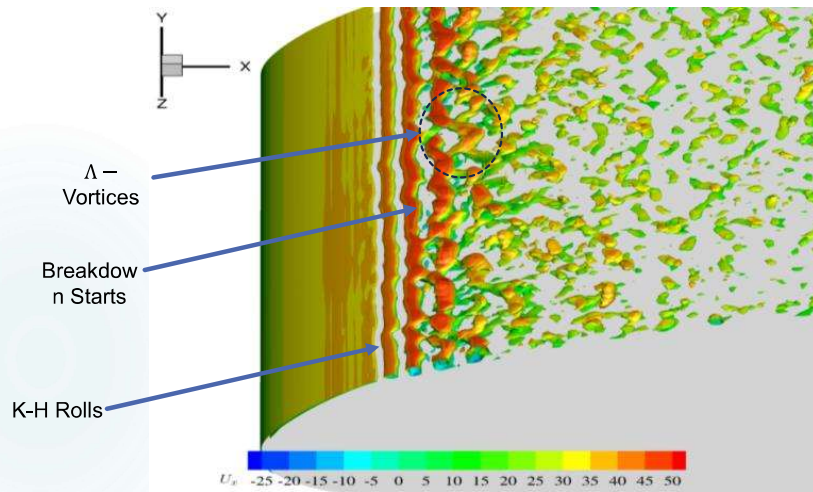


# Flow Transition

# Q – Iso-Surfaces

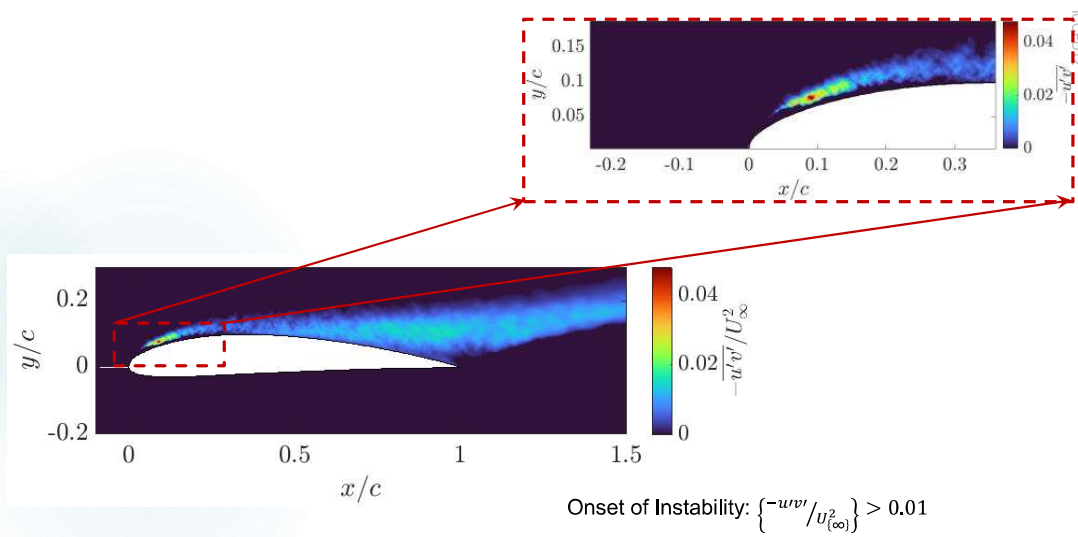


# K-H Roll Breakdown



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# Flow Transition



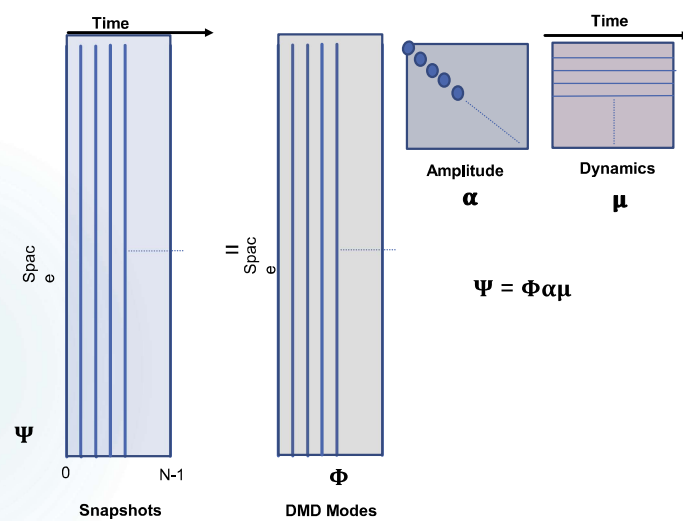
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## Unsteady Flow Analysis

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## Dynamic Mode Decomposition

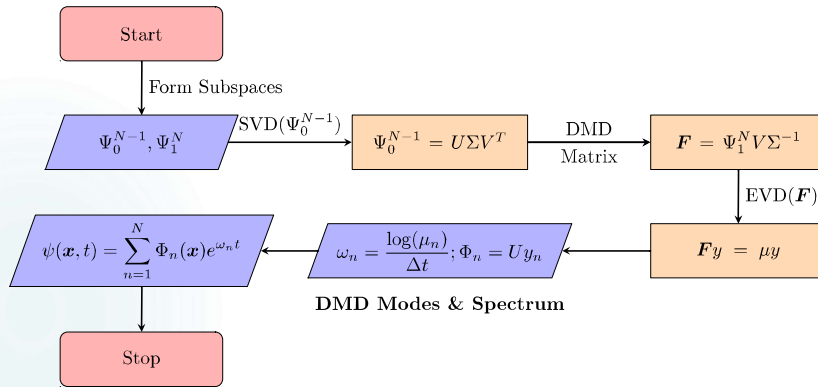


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# DMD Algorithm

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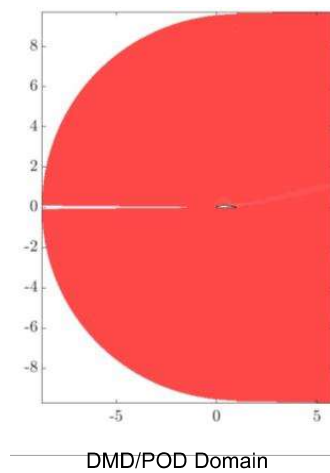
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# Dynamic Mode Decomposition (DMD)

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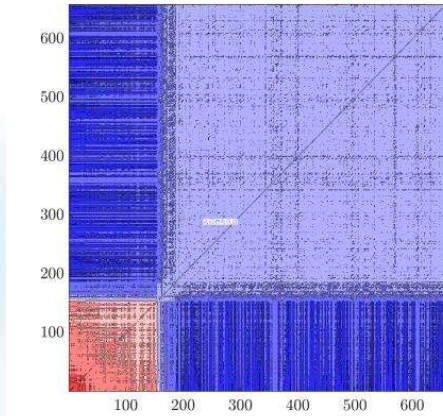
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- DMD performed on the midplane of LES snapshots
- Velocity (u & v) variables are chosen for the analysis
- Snapshots are taken after discarding initial transient datasets
- DMD convergence is obtained by varying number of snapshots
- DMD performed on a reduced domain

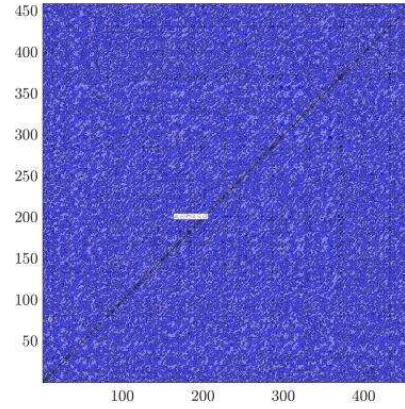


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# DMD – Selection of Snapshots

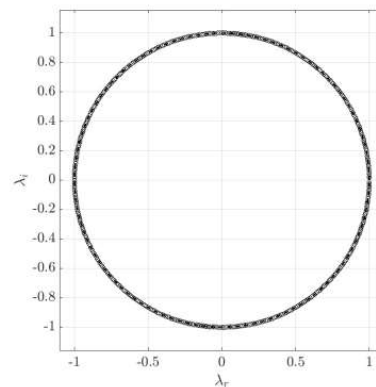
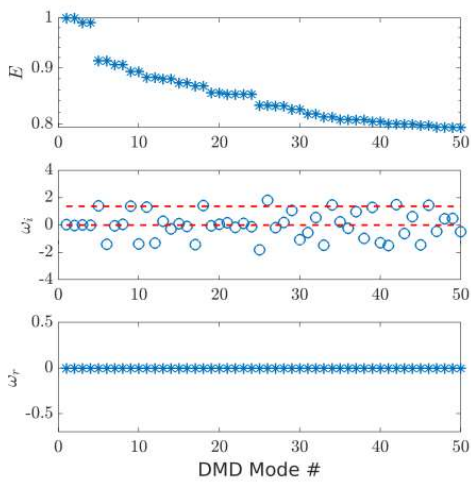


Initial



After removing Transients

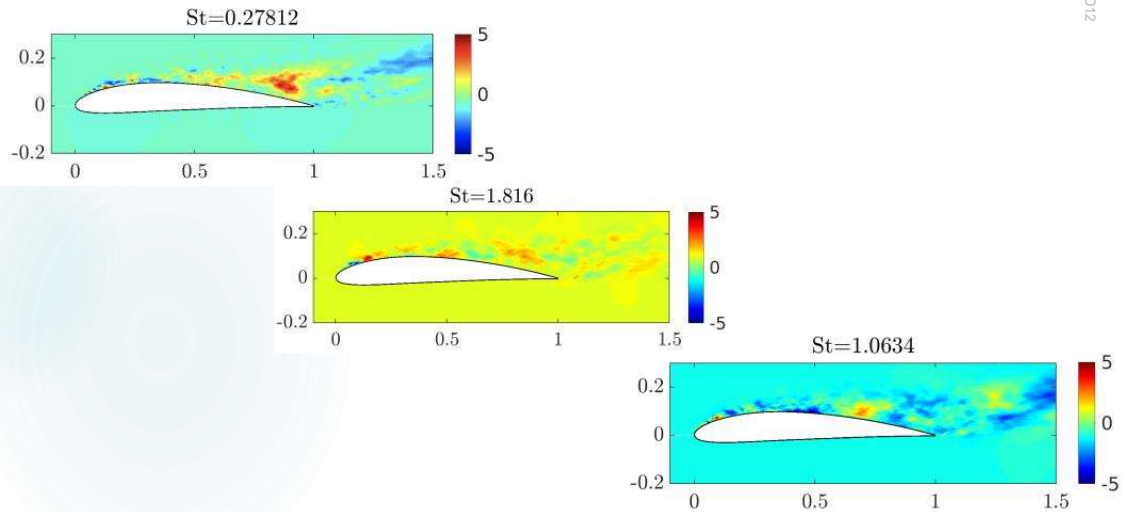
# Dynamic Mode Decomposition (DMD)



## DMD Modes

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## Proper Orthogonal Decomposition (POD)

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Method of Snapshots

$$\mathbf{X} = \begin{bmatrix} | & | & \cdots & | \\ u_1 & u_2 & \cdots & u_N \\ | & | & \cdots & | \end{bmatrix}$$

Formation of covariance matrix

$$\mathbf{C} = \frac{1}{N} \mathbf{X} \mathbf{X}^T$$

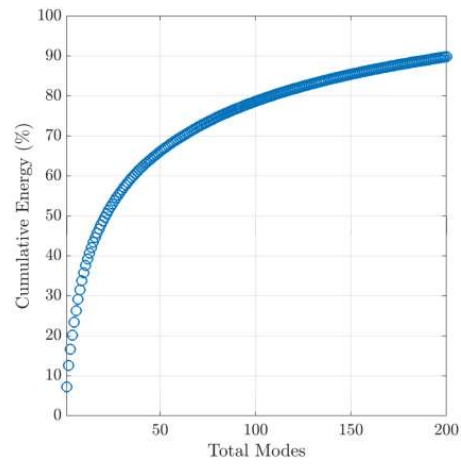
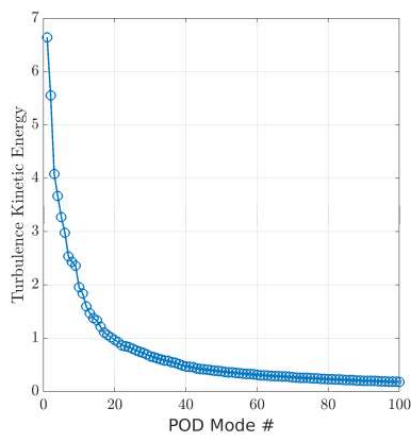
Eigenvalue Decomposition

$$\mathbf{C} \mathbf{v}_i = \lambda_i \mathbf{v}_i$$

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# Proper Orthogonal Decomposition (POD)

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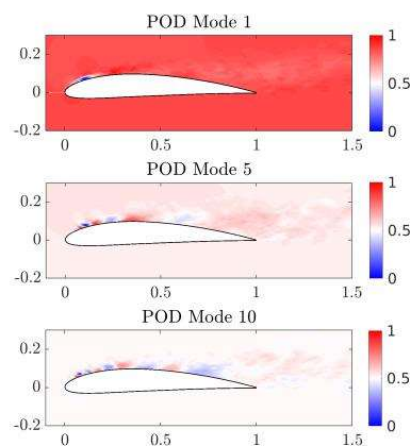


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# Proper Orthogonal Decomposition (POD)

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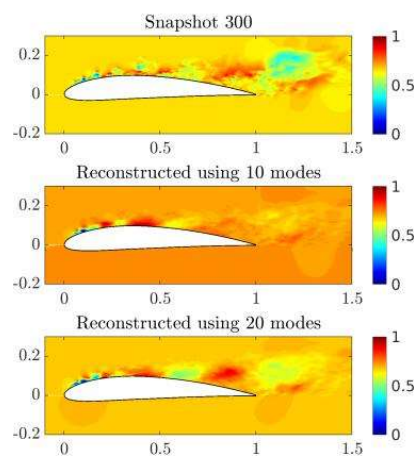
## Attempt to ROM

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## Reconstruction using POD modes

10 Modes: About 35% of TKE

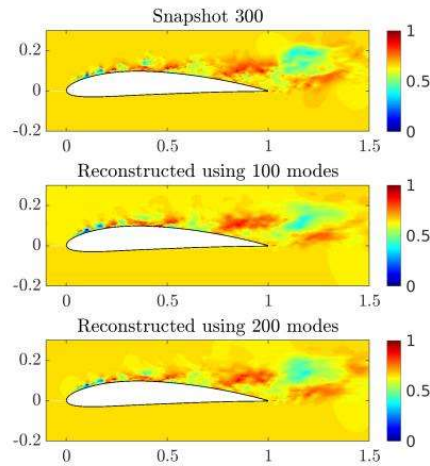
20 Modes: About 48% of TKE



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100 Modes: About 78% of TKE

200 Modes: About 90% of TKE



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## Conclusions

RANS, URANS and LES are performed on NACA4412 wing at  $14^\circ$  angle of attack

LES with dynamical  $k$  -equation predicts the overall profile well except at the trailing edge, where a smaller separation bubble is predicted

Finer flow structures reveals transition-to-turbulence via K-H instability on the suction side  $\frac{x}{c} = 0.05$ . However, the transition zone is small due to high Reynolds number.

POD analysis suggests that a high-rank system with first 10 modes constituting only 35% of the total kinetic energy.

Reconstruction with about 10 modes shows the K-H modes as well as wake region. However, reproduction of finer structures require more than 200 modes.

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# Acknowledgments

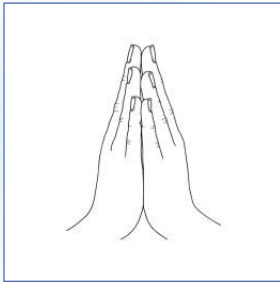
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IIT Kanpur Supercomputing Resources (Param Sanganak)

IIT Kanpur Initiation Grant



Thank you very much!

Contact: [rajeshr@iitk.ac.in](mailto:rajeshr@iitk.ac.in)