Oral presentation | Reduced order models

Reduced order models-II

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[2-C-01] Model Order Reduction by Convex Displacement Interpolation

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Model order reduction by convex displacement interpolation

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1 Introduction

We will initially present a nonlinear interpolation technique for parametric fields that exploits optimal transportation of coherent structures of the solution to achieve accurate performance. The approach generalizes the nonlinear interpolation procedure introduced in [1] to multi-dimensional parameter domains and to datasets of several snapshots. Given a library of high-fidelity simulations, we rely on a scalar testing function and on a point set registration method to identify coherent structures of the solution field in the form of sorted point clouds. Given a new parameter value, we exploit a regression method to predict the new point cloud; then, we resort to a boundary-aware registration technique to define bijective mappings that deform the new point cloud into the point clouds of the neighboring elements of the dataset, while preserving the boundary of the domain; finally, we define the estimate as a weighted combination of modes obtained by composing the neighboring snapshots with the previously-built mappings.

In comparison to the recent development in [2], we will outline a new and effective, easily implementable methodology to determine bijective mappings that respect the domain boundary. This approach results in an accurate non-linear interpolation of solutions for parameter values not explored previously.

2 Problem Statement

Despite the many recent contributions, model order reduction (MOR) of parametric problems with compactly-supported features — such as shocks or shear layers — remains an outstanding task for state-of-the-art techniques due to the fundamental inadequacy of linear approximations. The aim of our approach is to devise a general — i.e., independent of the underlying equations — interpolation technique for steady-state parametric problems, with emphasis on fluid dynamics applications.

During the past decade, several authors have proposed *mapping methods* to deal with this class of problems [3, 4, 5, 6, 7, 8, 9, 10, 11]. We denote by μ the vector of model parameters in the region $\mathcal{P} \subset \mathbb{R}^p$ and we denote by $\Omega \subset \mathbb{R}^d$ the open computational domain; then, we introduce the parametric field of interest $u_\mu : \Omega \times \mathcal{P} \to \mathbb{R}^D$ and the solution manifold $\mathcal{M} = \{u_\mu := u(\cdot;\mu) : \mu \in \mathcal{P}\}$. Lagrangian approximations rely on the ansatz:

$$\widehat{u}_\mu = \widetilde{u}_\mu \circ \Phi_\mu^{-1}, \tag{1a}$$

where $\widetilde{u}_\mu$ is a linear (or affine) approximation of the form

$$\widetilde{u}_\mu(x) = \sum_{i=1}^{n} \widehat{\omega}_\mu^i \zeta_i(x), \quad x \in \Omega, \mu \in \mathcal{P}, \tag{1b}$$

for proper choices of the weights $\widehat{\omega}_\mu^1, \ldots, \widehat{\omega}_\mu^n$ and the parameter-independent fields $\zeta_1, \ldots, \zeta_n : \Omega \to \mathbb{R}^D$, and $\Phi : \Omega \times \mathcal{P} \to \Omega$ is a suitably-chosen bijection that tracks the coherent structures of the solution; here, D denotes the number of state variables, while d is the spatial dimension. Lagrangian approaches are motivated by the observation that in many problems of interest coherent structures that are troublesome for linear approximations vary smoothly with the parameter and they hence can be tracked through a low-rank parameter-dependent mapping Φ.

In [1], a general method dubbed convex displacement interpolation (CDI) was proposed. It relies on optimal transportation to perform accurate nonlinear interpolations between solution snapshots; the approach was developed for databases of two snapshots and one-dimensional parameter domains. Similarly to Lagrangian approaches, CDI relies on the assumption that the location of coherent features of the solution field depends smoothly on the parameter; however, unlike in (1), it does not rely on the definition of a reference configuration where the location of the coherent features is (approximately) freezed.

In our presentation, we will explain CDI and delve into its recent expansion into multi-dimensional parameter domains and datasets with multiple snapshots, as discussed in [2]. Additionally, we will introduce a novel method utilizing a one-shot optimization technique to establish a bijective parameter-dependent mapping Φ that provides accurate, non-intrusive interpolation of distributed fields.

We will illustrate the accuracy of the method through various numerical examples, encompassing compressible and incompressible, viscous and inviscid flows. Moreover, we will demonstrate how to utilize the nonlinear interpolation procedure to enhance the simulation dataset for linear-subspace projection-based model reduction.

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