Oral presentation | Reduced order models

Reduced order models-II

Mon. Jul 15, 2024 2:00 PM - 4:00 PM Room C

[2-C-02] A hybrid-fidelity model for floating wave energy converters

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Keywords: Multi-fidelity numerical model, Wave energy converter, Domain decomposition

A hybrid-fidelity model for floating wave energy converters

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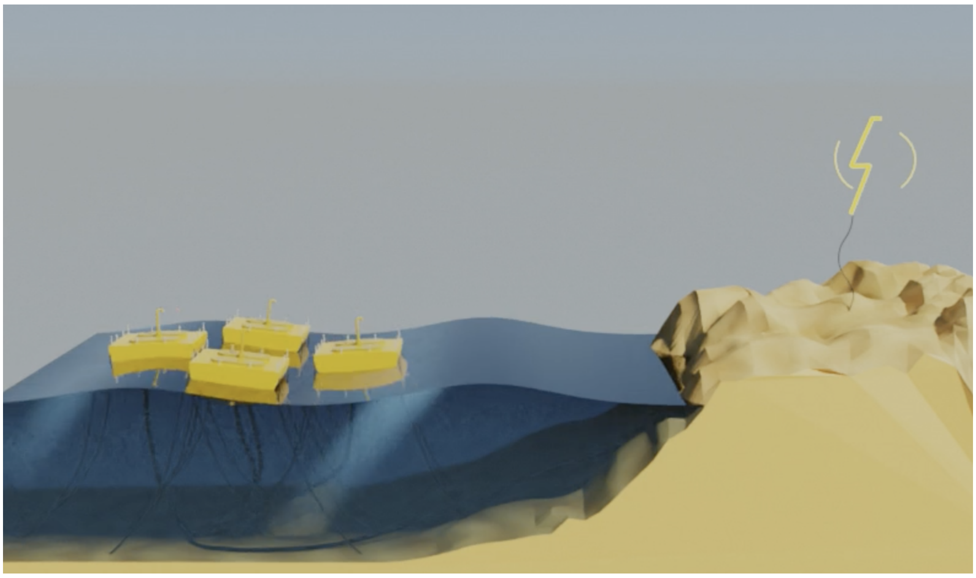
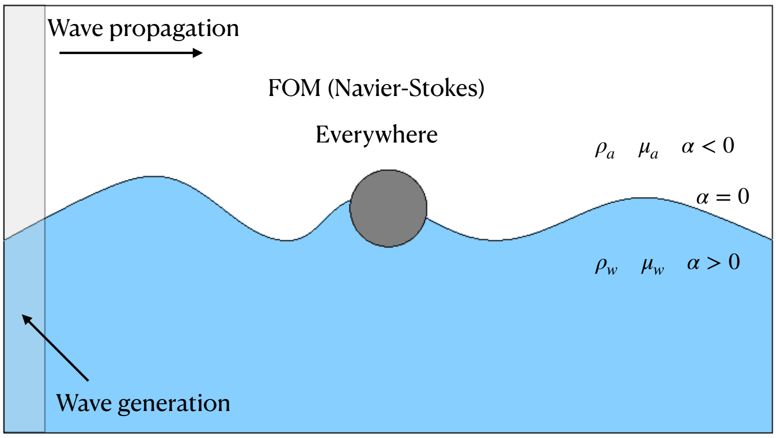
ICCFD, Kobe, Japan, July 2024

General problem and flow configuration



Source: Politecnico di Torino & Marine Offshore renewable energy

General problem and flow configuration



- **Incompressible bi-fluid Navier-Stokes equations (to avoid surface fitted-grids)**

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u} = -\frac{1}{\rho}\boldsymbol{\nabla} p + \frac{1}{\rho}\boldsymbol{\nabla}\cdot\mu\left(\boldsymbol{\nabla}\boldsymbol{u} + (\boldsymbol{\nabla}\boldsymbol{u})^T\right) + \boldsymbol{g},$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{u} = 0,$$

$$\frac{\partial \alpha}{\partial t} + \boldsymbol{u}\cdot\boldsymbol{\nabla}\alpha = 0 \;\Rightarrow\; \rho = \rho_a + (\rho_w - \rho_a)H(\alpha),\; \mu = \mu_a + (\mu_w - \mu_a)H(\alpha)$$

$+$ IC and BCs (wave imposed on the gray zone).

↪ **Too costly! ⇒ we can only afford few numerical simulations! (how to select??)**

Need to reduce the CPU costs

The Full Order Models are almost never used alone for wind or marine energy applications ⇒ large CPU costs, multiscale problem, optimization

- **Simplified mathematical models (invariance-asymptotic)**
  - ↪ Shallow Water Equations, Boussinesq, etc (post doc Umberto Bosi)
  - ↪ Inviscid incompressible Navier-Stokes equations (PhD Caroline Le Guern)

- **Simplified numerical models based on data (only polar curves for 2D airfoils)**
  - ↪ Actual blades are modeled using extra Volume Forces based on data
  - ↪ Actuator lines: (post doc Nishant Kumar)

- **Model Order Reduction based on data (primitive variables)**
  - ↪ Proper Orthogonal Decomposition Reduced Order Model (PhD Beatrice Battisti)

$$\boldsymbol{U}(\boldsymbol{x},\, t) = \sum_{i=1}^{N} a_i(t)\boldsymbol{\Phi}_i(\boldsymbol{x}) \quad U = u, v, w, p, \rho, \mu, \ldots$$

Low dimensional subspace

Proper Orthogonal Decomposition (POD), Lumley (1967)

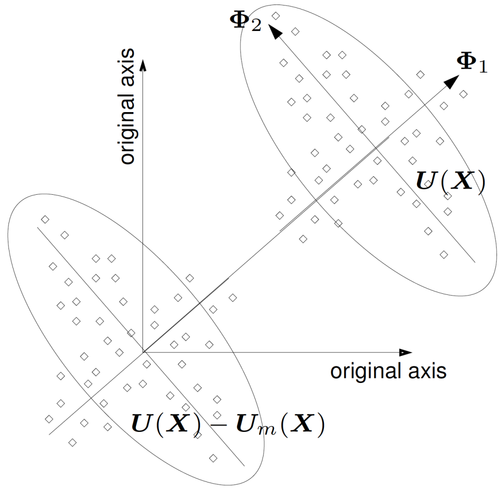
▷ Look for the flow realization $\Phi(X)$ that is "the closest" in an average sense to realizations $U(X)$.

$$(X = (x, t) \in \mathcal{D} = \Omega \times \mathbb{R}^+)$$

▷ $\Phi(X)$ solution of problem:

$$\max_{\Phi} \langle |(U, \Phi)|^2 \rangle, \quad \|\Phi\|^2 = 1.$$

▷ Optimal convergence *in L^2 norm* de $\Phi(X)$
⇒ Dynamical reduction possible.



Lumley J.L. (1967) : The structure of inhomogeneous turbulence. *Atmospheric Turbulence and Wave Propagation*, ed. A.M. Yaglom & V.I. Tatarski, pp. 166-178.

Low dimensional subspace

▷ Equivalent with Fredholm equation, $R(X, X')$ is *space-time correlation tensor*

$$\int_{\mathcal{D}} R_{ij}(X, X')\Phi_n^{(j)}(X')\, dX' = \lambda_n \Phi_n^{(i)}(X) \qquad n = 1, .., N_s$$

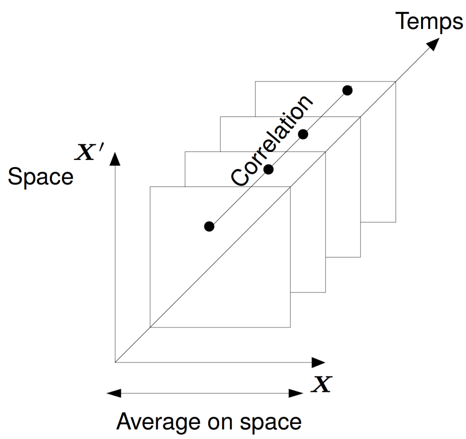
▷ Snapshots method, Sirovich (1987) :

$$\int_T C(t, t')a_n(t')\, dt' = \lambda_n a_n(t)$$

▷ POD basis $\Phi(X)$ with N_s snapshots

$$U(x, t) = \sum_{n=1}^{N_s} a_n(t)\Phi_n(x),$$

$$\widetilde{U}(x, t) = \sum_{n=1}^{N_r} a_n(t)\Phi_n(x), \quad \text{with} \quad N_r \ll N_s.$$



▷ **POD basis $\Phi(X)$ highly depends on the snapshots (sampling problem)**

Sirovich L. (1987) : Turbulence and the dynamics of coherent structures. Part 1,2,3 *Quarterly of Applied Mathematics*, **XLV** N° 3, pp. 561–571.

Full Order Model and POD reduced order model

- **A POD ROM in the whole computational domain?**

↪ How to deal with complex body deformations and motions?
↪ Is a single POD ROM accurate in the whole domain?
↪ Is the same accuracy necessary in the whole domain?

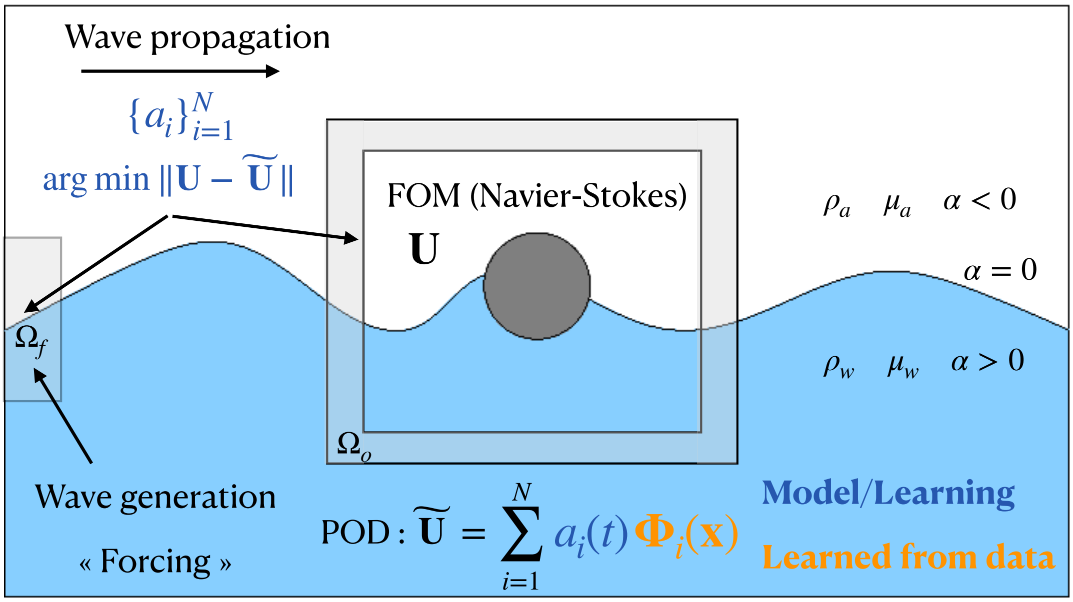
- **Past observations, for academic to industrial configurations**

↪ Large POD projection errors in the vicinity of the obstacles
↪ Low POD projection errors elsewhere

- **Proposed solution**

↪ Couple FOM in the vicinity of the obstacles with POD ROM elsewhere

General configuration



↪ The POD basis functions $\{\mathbf{\Phi}_i\}_{i=1}^N$ are **learned from data** (2nd part of this talk)
↪ The POD coefficients $\{a_i\}_{i=1}^N$ can be obtained by **optimization (Galerkin-free)**

Reduced Order Model

Generalized coordinates $\{a\}_{i=1}^{N_r}$

(a) (Petrov-) Galerkin Reduced Order Model ($N_r \ll N_s$)

$$\boldsymbol{U} = \boldsymbol{U}_g + \sum_{n=1}^{N_r} a_n \boldsymbol{\Phi}_n, \quad \text{POD on "scaled" } \boldsymbol{U} = \left(\boldsymbol{u}, \frac{1}{\rho}, \mu, p\right)^T$$

$$\left(\boldsymbol{\Phi}_i^u, \frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{u}\right) = \left(\boldsymbol{\Phi}_i^u, -\frac{1}{\rho}\boldsymbol{\nabla} p + \frac{1}{\rho}\nabla \cdot \mu(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T)\right).$$

▶ Dynamical system

$$\begin{cases} \dfrac{d\, a_i(t)}{d\, t} = \mathcal{A}_i + \displaystyle\sum_{j=1}^{N_r} \mathcal{B}_{ij}\, a_j(t) + \sum_{j=1}^{N_r}\sum_{k=1}^{N_r} \mathcal{C}_{ijk}\, a_j(t) a_k(t) + \sum_{j=1}^{N_r}\sum_{k=1}^{N_r}\sum_{l=1}^{N_r} \mathcal{D}_{ijkl}\, a_j(t) a_k(t) a_l(t) \\ a_i(0) = (\boldsymbol{u}(\boldsymbol{x}, 0), \boldsymbol{\Phi}_i(\boldsymbol{x})). \end{cases}$$

↪ Costly ⇒ The 4^{th} order tensor involved is to costly to build and to solve!)
- ↪ If number of modes is 100 ⇒ size of 100 millions...
- ↪ Not compatible with Model Order Reduction
- ↪ Hyperreduction: Not compatible with "industrial" numerical solver

Reduced Order Model

Generalized coordinates $\{a\}_{i=1}^{N_r}$

(b) Galerkin-free Reduced Order Model

What variables? ⇒ whose are measured at inflow AND required for FOM BCs

$$\text{Velocity:} \quad \widetilde{\boldsymbol{u}} = \boldsymbol{u}_g + \sum_{i=1}^{N_r} \hat{u}_i \boldsymbol{\Phi}_i,$$

$$\text{Color function (VOF, LS):} \quad \widetilde{\alpha} = \alpha_g + \sum_{i=1}^{N_r} \hat{\alpha}_i \Psi_i \quad \Rightarrow \rho,\ \mu.$$

The functions $\boldsymbol{u}_g$ and α_g can be snapshots average, or any desired functions

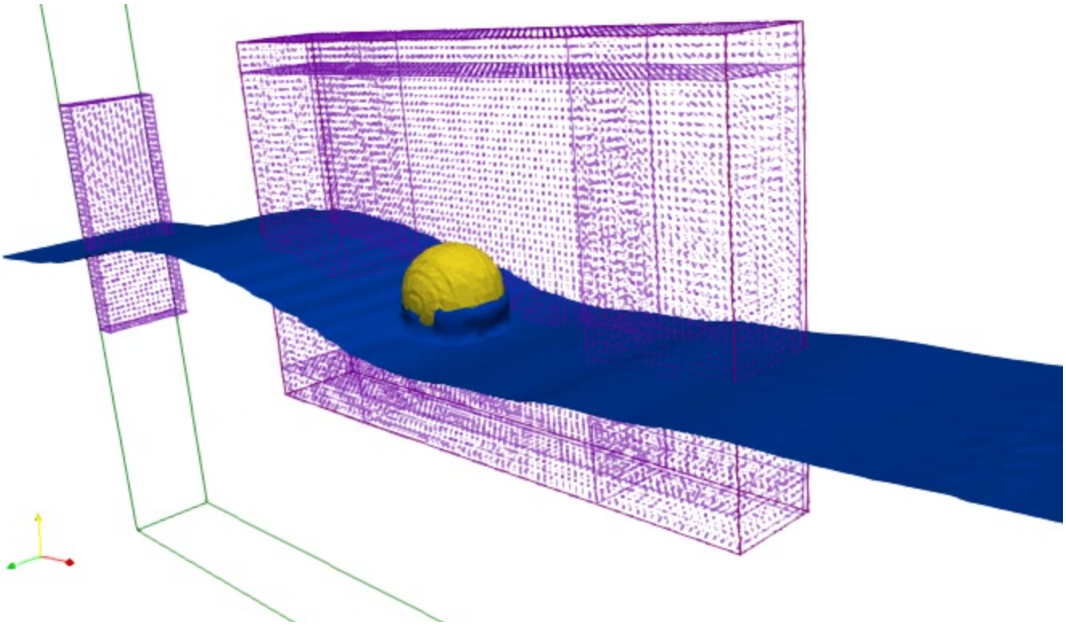
↪ $\{\hat{u}\}_{i=1}^{N_r}$ ← Least squares minimization of $\|\boldsymbol{u}_h - \widetilde{\boldsymbol{u}}\|_2$ in "gray" domains $\Omega_o \cup \Omega_f$,

↪ $\{\hat{\alpha}\}_{i=1}^{N_r}$ ← Least squares minimization of $\|\alpha_h - \widetilde{\alpha}\|_2$ in "gray" domains $\Omega_o \cup \Omega_f$.

↪ More stable than classical Galerkin projection since HD informations are involved

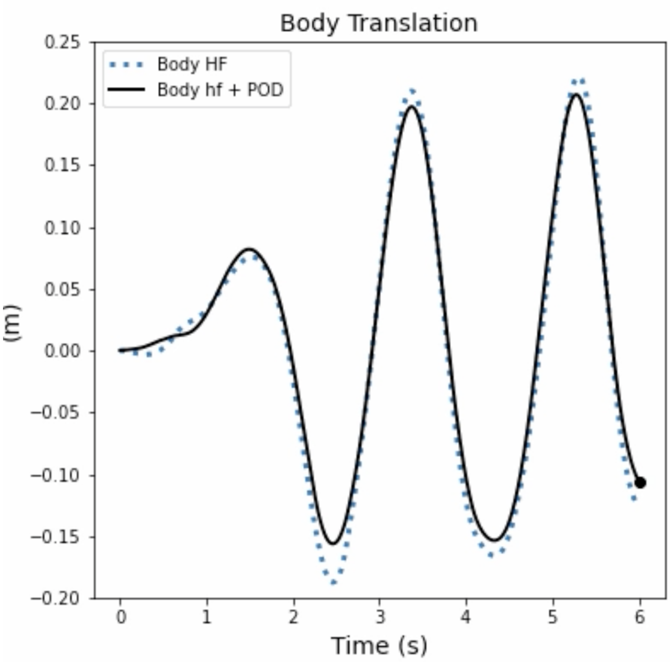
In any case, an adapted POD subspace is required!!

Example for sea wave energy converter (point absorber)



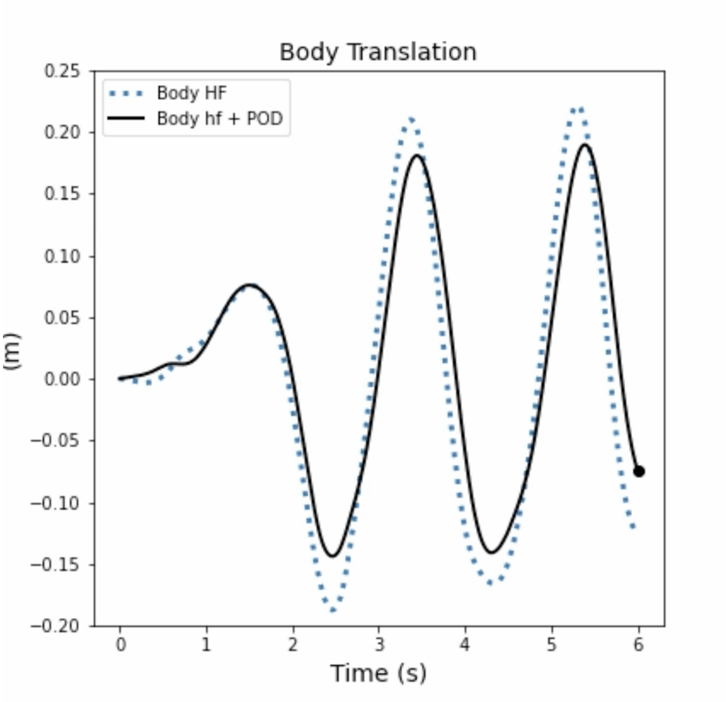
In sample ("reproduction" with $N_r = 30$ modes)
POD basis Φ built using snapshots from exact wave

Example for sea wave energy converter (point absorber)



Out-of-sample ("prediction" with $N_r = 30$ modes)
POD basis Φ built using snapshots from two "nearby" waves (in parameter space)

Example for sea wave energy converter (point absorber)



Out-of-sample ("prediction" with $N_r = 30$ modes)
POD basis Φ built using snapshots from two "distant" waves (in parameter space)

Distances in the solution space

Computation of $\{\Phi_n\}_{n=1}^{N_r}$
$\Rightarrow$ A robust POD subspace is required!

- **How to perform an efficient sampling of input parameter space?**

$\hookrightarrow$ Previous studies: Uniform Sampling in a Cartesian way. Problem: not optimal
$\hookrightarrow$ distance in parameter space $\neq$ "distance" in solution space

- **Iterative sampling based on an error criterion (OLD)**

$\hookrightarrow$ Iterative method to improve the POD basis
$\hookrightarrow$ The error is the mathematical projection error computed using the current POD basis
$\hookrightarrow$ "Adaptive mesh refinement" using Delaunay triangulation (dual of Voronoi tesselation)

- **Iterative sampling based on a distance criterion (NEW)**

$\hookrightarrow$ Distance between solution: steady vs. unsteady

Distances in the solution space

▶ **Steady problems**

↪ Computing the distance between steady solutions for different operating conditions is "easy" (Wasserstein distance, relative difference/error)

▶ **Unsteady problems**

↪ Computing the distance between solutions (set of snapshots) for different operating conditions for unsteady problems is not straightforward

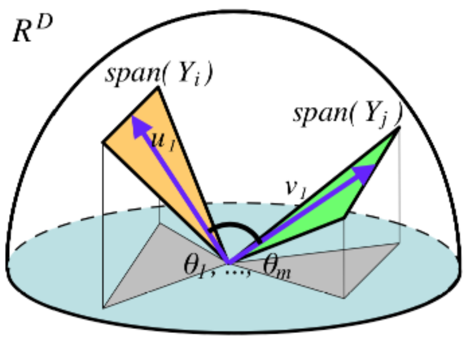
↪ Easier to compute distance between subspaces spanned by solutions (POD basis) for different operating conditions

↪ Geodesics on the Grassmann manifold / Principal Angles with other metrics

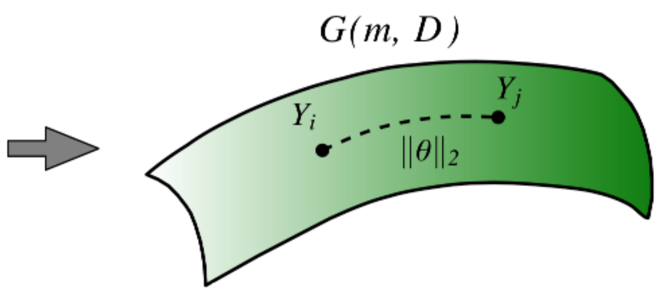
Distances in the solution space

↪ We thus consider two POD basis $\mathbf{\Phi}_1 \in \mathbb{R}^{Nf \times N_\Phi}$ and $\mathbf{\Phi}_2 \in \mathbb{R}^{Nf \times N_\Phi}$.

(The columns of $\mathbf{\Phi}_i$ provides a basis of a subspace $\mathcal{S}_i$ of dimension N_Φ in $\mathbb{R}^{N_f}$)



Principal Angles



Geodesic on the Grassmann manifold

↪ Interpolations using angles or along the geodesic are possible

↪ **Be sure the solution to be interpolated is on (or close to) the geodesic!!**

↪ **Interpolation should be performed using "quite close" points**

Distances in the solution space

Principal Angles Between Subspaces (PABS)

(J. Hamm & D.D. Lee, Grassmann Discriminant Analysis)

$\hookrightarrow$ Definition: the principal angles $0 \leq \theta_1 \leq \theta_2 \leq \cdots \leq \theta_{N_\Phi} \leq \frac{\pi}{2}$ between two subspaces span($\mathbf{\Phi}_1$) and span($\mathbf{\Phi}_2$), are defined recursively by

$$\cos\theta_k = \max_{\boldsymbol{u}_k \in \text{span}(\mathbf{\Phi}_1)} \max_{\boldsymbol{v}_k \in \text{span}(\mathbf{\Phi}_2)} \boldsymbol{u}_k' \boldsymbol{v}_k,$$

$$\text{subject to} \quad \boldsymbol{u}_k'\boldsymbol{u}_k = \boldsymbol{v}_k'\boldsymbol{v}_k = 1, \quad \text{and} \quad \boldsymbol{u}_k'\boldsymbol{u}_i = \boldsymbol{v}_k'\boldsymbol{v}_i = 0 \quad (i = 1, \ldots, k-1)$$

$\hookrightarrow$ Practical computation via SVD:

$$\mathbf{\Phi}_1'\mathbf{\Phi}_2 = \boldsymbol{U}(\cos(\boldsymbol{\theta}))\boldsymbol{V}'$$

with $\boldsymbol{U} = [\boldsymbol{u}_1 \ \ldots \ \boldsymbol{u}_{N_\Phi}]$, $\boldsymbol{V} = [\boldsymbol{v}_1 \ \ldots \ \boldsymbol{v}_{N_\Phi}]$ and $\cos\boldsymbol{\theta} = \text{diag}(\cos\theta_1 \ \ldots \ \cos\theta_{N_\Phi})$

Robustness of the POD subspace | Sampling of the input parameter space

Distances in the solution space

Principal Angles Between Subspaces (PABS)

(J. Hamm & D.D. Lee, Grassmann Discriminant Analysis)

$\hookrightarrow$ Different Metrics are usually used

– Projection: $d_P(\mathcal{S}_1, \mathcal{S}_2) = \left(\sum_{i=1}^{N_\Phi} \sin^2\theta_i\right)^{\frac{1}{2}}$

– Binet-Cauchy: $d_{BC}(\mathcal{S}_1, \mathcal{S}_2) = \left(1 - \prod_i \cos^2\theta_i\right)^{\frac{1}{2}}$

– Max-Min Correlation: $d_{Max}(\mathcal{S}_1, \mathcal{S}_2) = \sin\theta_1, \quad d_{Min}(\mathcal{S}_1, \mathcal{S}_2) = \sin\theta_{N_\Phi}$

– Grassmann distance: $d_G(\mathcal{S}_1, \mathcal{S}_2) = \left(\sum_{i=1}^{N_\Phi} \theta_i^2\right)^{\frac{1}{2}}$

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Distances in the solution space

Geodesic on the Grassmann manifold

(D. Amsallem and C. Farhat, AIAA Journal, 2008)

– The subspace $\mathcal{S}_i = \text{span}(\boldsymbol{\Phi}_i)$ belongs to the Grassmann manifold $\mathcal{G}(N_\Phi, N_f)$
$\hookrightarrow$ $\mathcal{G}(N_\Phi, N_f)$ is defined as the set of all N_Φ-dimensional subspaces of $\mathbb{R}^{N_f}$

– Each N_Φ-dimensional subspaces of $\mathbb{R}^{N_f}$ can be viewed as a point on $\mathcal{G}(N_\Phi, N_f)$

– It is thus possible to define distance $d_G(\mathcal{S}_1, \mathcal{S}_2)$, the geodesic between these points.

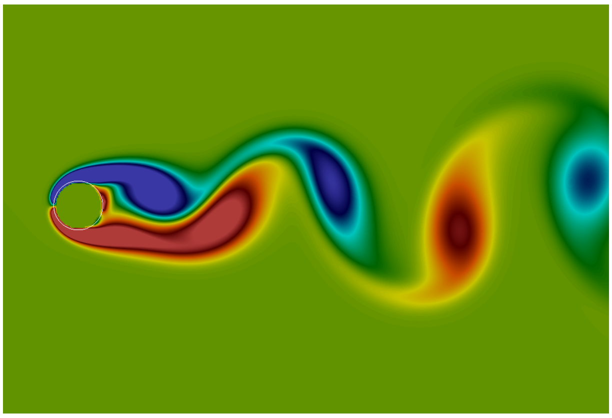
$\hookrightarrow$ Practical computation via thin SVD:

$$(\boldsymbol{I} - \boldsymbol{\Phi}_1\boldsymbol{\Phi}_1')\boldsymbol{\Phi}_2(\boldsymbol{\Phi}_1'\boldsymbol{\Phi}_2) = \boldsymbol{U}\Sigma\boldsymbol{V}' \quad \text{with} \quad \boldsymbol{\theta} = \tan^{-1}(\boldsymbol{\Sigma})$$

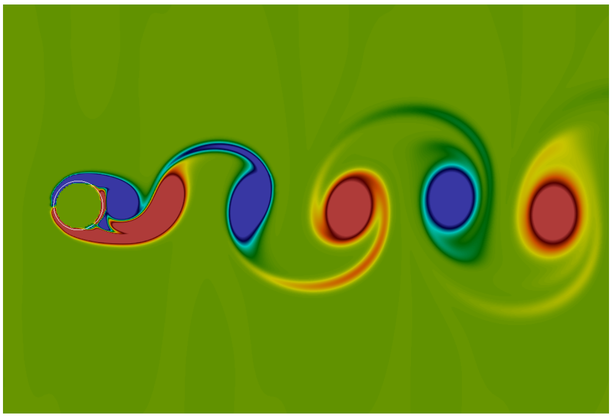
$\hookrightarrow$ Can be used to perform interpolation between more than two subspaces

Configuration

- **Goal**
  - $\hookrightarrow$ We want to predict the flow characteristics for $100 \leq Re \leq 500$
  - $\hookrightarrow$ For low numerical costs



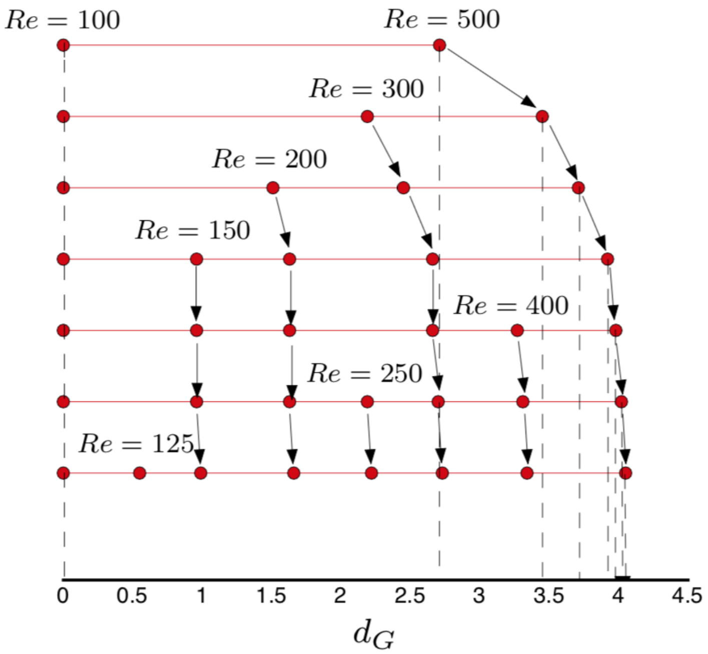
$Re = 100$



$Re = 500$

Sampling

▶ **Sampling by continuation method on the Grassmann geodesic**



$Re = 125$ is almost on the geodesic $Re = 100 \leftrightarrow Re = 150$!

Almost uniform sampling **in the solution space**!!

Subspaces interpolation

▶ **Interpolation for one dimensional input parameter space between s_0 and s_1**

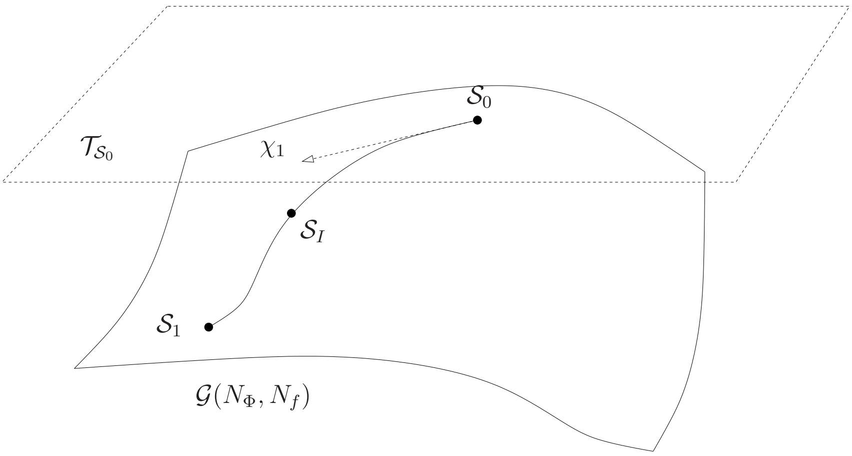
↪ On the geodesic between two points on the Grassmann manifold $\mathcal{S}_0$ and $\mathcal{S}_1$

↪ $\mathcal{T}_{\mathcal{S}_0}$ is the tangent space to the Grassmann manifold at $\mathcal{S}_0$

↪ χ_1 is the geodesic initial condition given by $\Gamma = \boldsymbol{U}\tan^{-1}(\boldsymbol{\Sigma})\boldsymbol{V}'$ on $\mathcal{T}_{\mathcal{S}_0}$

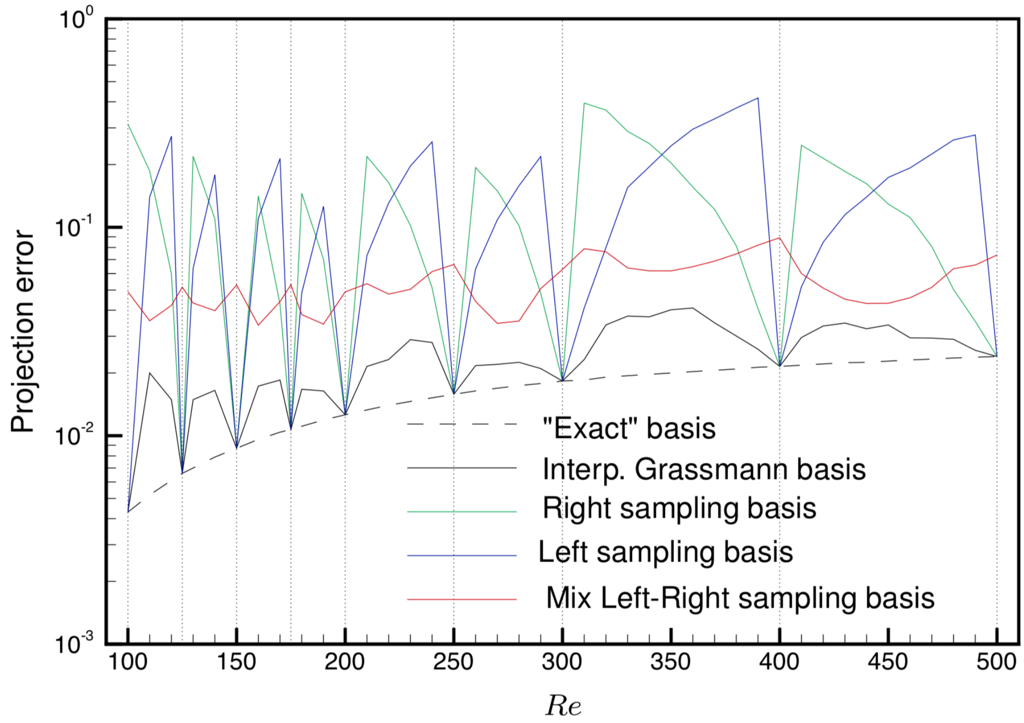
where $(\boldsymbol{I} - \boldsymbol{\Phi}_0\boldsymbol{\Phi}_0')\boldsymbol{\Phi}_1(\boldsymbol{\Phi}_0'\boldsymbol{\Phi}_1) = \boldsymbol{U}\Sigma\boldsymbol{V}'$ with $\boldsymbol{\theta} = \tan^{-1}(\boldsymbol{\Sigma})$

↪ Interpolation $\boldsymbol{\Phi}(s) = \text{span}\left[\boldsymbol{\Phi}_0\boldsymbol{V}\cos\left(\frac{s-s_0}{s_1-s_0}\boldsymbol{\theta}\right) + \boldsymbol{U}\sin\left(\frac{s-s_0}{s_1-s_0}\boldsymbol{\theta}\right)\right]$, $s \in [s_0, s_1]$



Subspaces interpolation

► **Evolution of L_2 errors for snapshots projection over different POD basis**



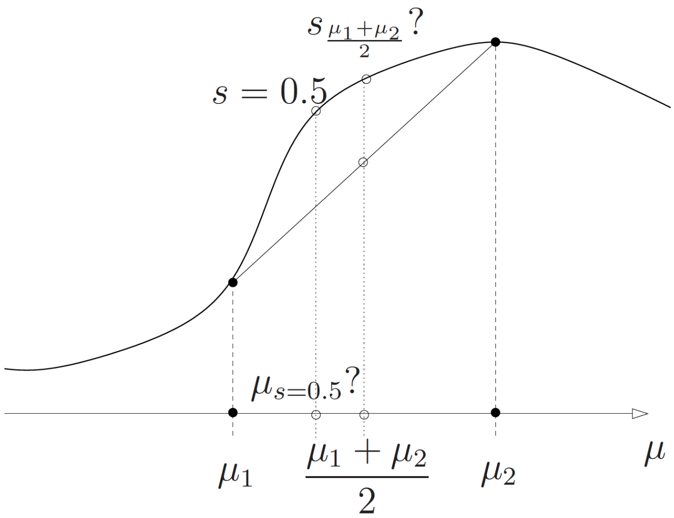
Comparison of projection errors on several basis

Improvement of the interpolation

► **Potential sources of errors**

↪ Computation of direction Γ_I from α and β is not adapted to the solution space

↪ Computation of s along direction Γ_I may not be adapted too



↪ What if one consider $\frac{1}{\mu}$ instead of μ? or generally $f(\mu)$? Where is the middle?

⇒ **Manifold learning to approximate appropriate $s = f(\mu)$ for interpolations!**

Conclusions & Perspectives

▶ **Sampling and interpolation**

- Sampling on Grassmann manifold or PABS
  ↪ Depends on the metric used (build on Principal Angles)
- Interpolation efficient in 1D input parameter space (almost linear)
  ↪ Fine and efficient sampling ⇒ piecewise manifold approximation is good
- Interpolation more difficult in 2D input parameter space
  ↪ Solution for $\frac{\mu_1+\mu_2}{2}$ may be not on the middle geodesic ($s = 0.5$)
  ↪ Interpolation parameters (α and β) should respect constrains
  ↪ Idea: try to approximate the geometry of the (Grassmann) solution manifold
  ↪ Isomap (shortest paths on distance graphs) + Multi-Dimensionnal Scaling

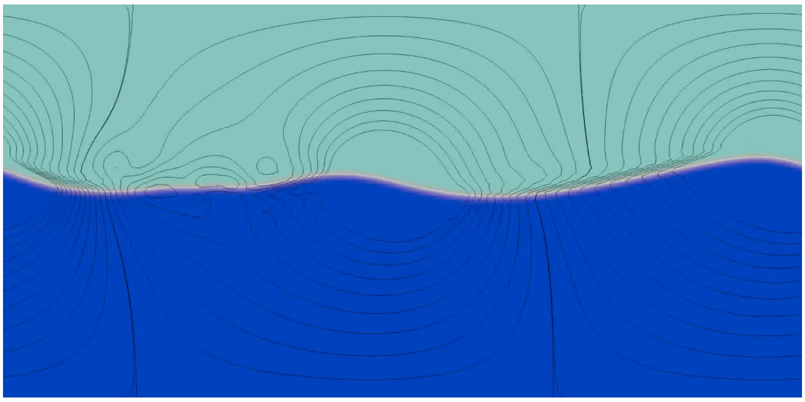
- Next: "dig" manifold approximation via Grassmann-MDS

Conclusions, perspectives and some remarks

Conclusions & Perspectives

▶ **Multi-fidelity numerical modeling: FOM and POD ROM coupling**

- Applied to renewable energy applications (WECs and wind-turbines)
  ↪ **"Problems" for WECs: POD of bi-fluid configurations not easy**
  Moving front: linear approx. not adapted...



- Snapshots clustering, then POD for each cluster (piecewise linear approx.)
- Snapshots mapping onto reference solution, then POD (non linear approx.)
  ↪ Optimal transport (non-linear approx.), quadratic approx. (Stanford)

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