Oral presentation | Multi-phase flow

Multi-phase flow-II

Mon. Jul 15, 2024 2:00 PM - 4:00 PM Room D

[2-D-03] Viscous Fingering coupled with Phase Separation

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Keywords: Viscous Fingering, Phase Separation, Phase-field Method

Viscous Fingering (VF) coupled with Phase Separation (PS)

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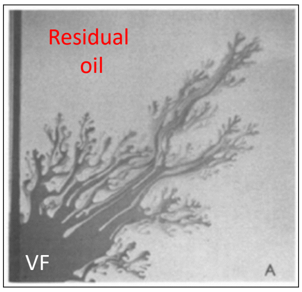
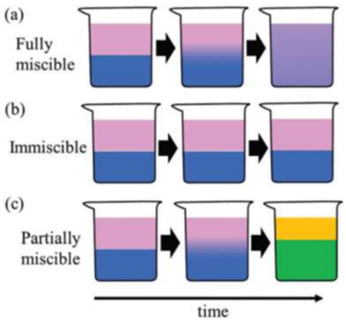
Underlying Instability

Viscous Fingering (VF) : Hydrodynamically unstable, i.e., more viscous fluid displaced by less viscous fluid in a porous medium



Miscible Five-Spot Displacement (Claridge, 1986)

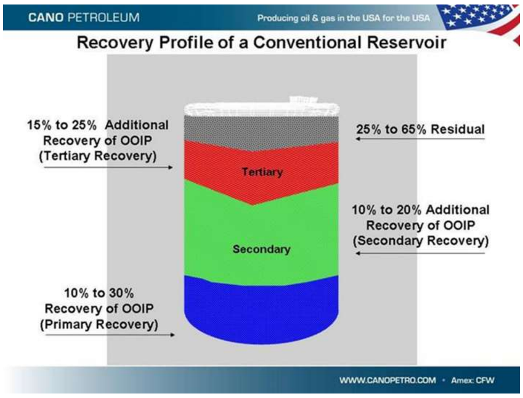


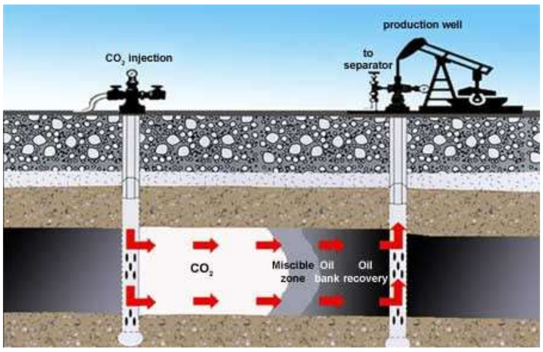
(Suzuki et al., Phys. Chem. Chem. Phys.,2021)

Phase Separation (PS) : Thermodynamically unstable, e.g., spinodal decomposition of metastable at particular composition

Enhanced Oil Recovery & Viscous Fingering

1, Interfacial Instability (Viscous Fingering) leads to less efficient oil recovery.
2, Suppress (control) of viscous fingering is a major issue to oil recovery efficiency.
3, CO2 is used as the displacing fluid to enhanced oil recovery (CO2 EOR).

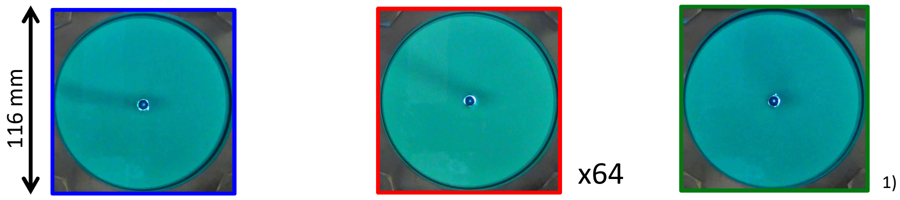




Solubility

✓Saffman-Taylor instability (Viscous fingering, VF) due to the viscosity contrast

Fully miscible	Immiscible	Partially miscible
Infinite solubility (Diffusion)	No solubility (Interfacial tension)	Finite (lower) solubility (Phase separation)



- ✓ Fully miscible and immiscible VFs show standard VF patterns.
- ✓ The partially miscible VF shows multiple droplets formation.
- ✓ This phenomenon cannot be explained by only hydrodynamic instability.

1) R. X. Suzuki et al., *J. Fluid Mech.*, 898, A11 (2020)

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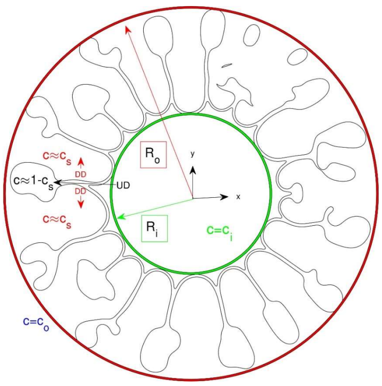
Motivation

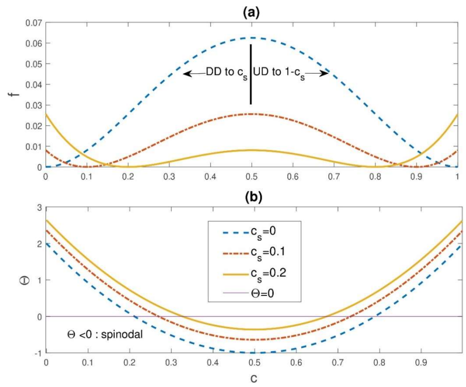
- ✓ **First** numerical study coupling effects of **hydrodynamic viscous fingering** and **thermodynamic phase separation** in a radial flow configuration
- ✓ Verify the **anomalous patterns**
- ✓ Comprehensive parametric studies to explain the underlying **mechanisms**



Uphill Diffusion (UD) toward miscibility (c_s)

&

Downhill Diffusion (DD) to complementary miscibility ($1-c_s$)



Governing Equations - Dimensionless

$$\nabla p = -\eta(c)\boldsymbol{u} - \frac{C}{I}\nabla \cdot [(\nabla c)(\nabla c)^T] \quad (1)$$

$$\frac{\partial c}{\partial t} + \boldsymbol{u} \cdot \nabla c = -\frac{1}{Pe}\nabla^2 \mu \quad (2)$$

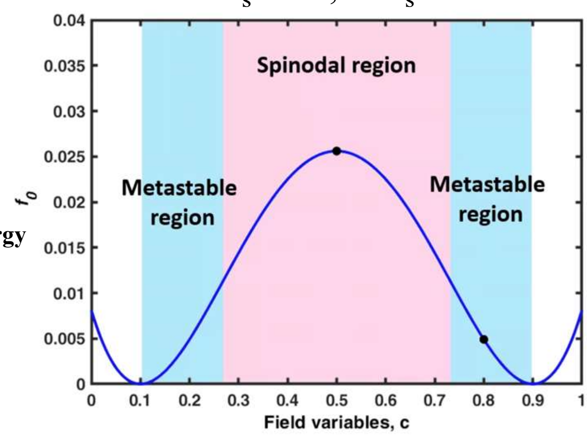
$$\mu = \frac{\partial f_0}{\partial c} + C\nabla^2 c \qquad \mu = \frac{\partial f_0}{\partial c} - C\epsilon\nabla^2 c. \quad (3)$$

$$f_0 = (c - cs)^2[c - (1 - cs)]^2$$

Spinodal : $\Theta = d^2 f_0/dc^2 < 0$

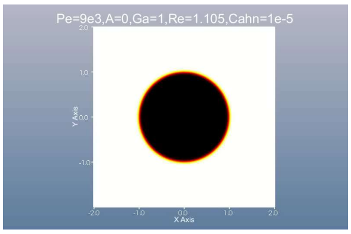
$c_s=0.1$; $1- c_s=0.9$

$\boldsymbol{u}$: velocity	$\eta(c)$: viscosity $\eta=e^{(1-c)R}$	f_0: **Helmholtz free energy**
p : pressure	C: Cahn number	μ : chemical potential
c : concentration	I: injection strength	Pe: Péclet number

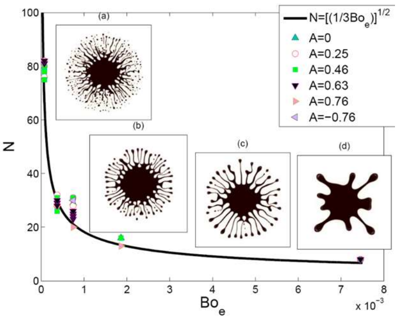


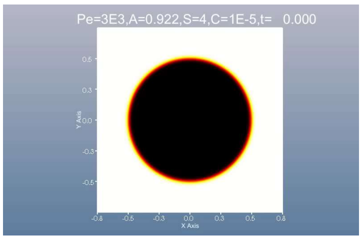
Numerical Schemes & Validation

- **Phase Concentration equation :**
  - 3^{rd} order Runge-Kutta procedure in time
  - 6^{th} order compact FD in space
- **Hele-Shaw equations : Vorticity-Streamfunction**
  - vorticity : 6^{th} order compact FD
  - Poisson (streamfunction) equation :
    y-direction - 6^{th} order compact FD
    x-direction pseudo-spectral scheme

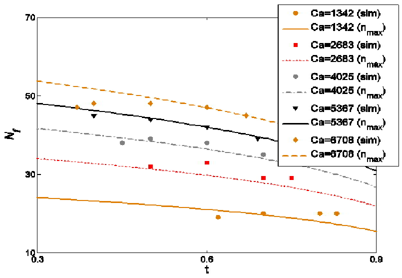


Immiscible & Rotational
Chen et al. PRE 2011



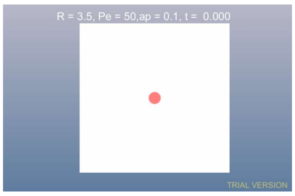


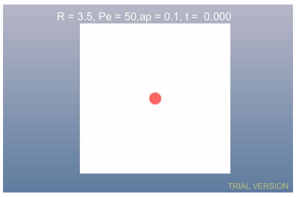
Immiscible & Suction
Chen et al. PRE 2014

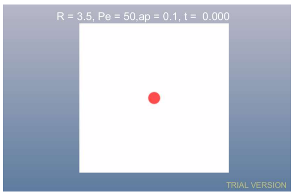


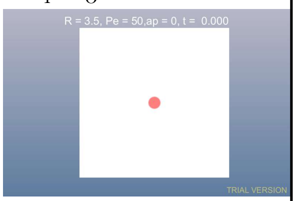
PS & VF w/o disturbance :R=3.5,Pe=50,I=12,C=10^{-5}

c_s= 0.1 c_i/c_o = 0.5/0.0	c_s= 0.1 c_i/c_o = 0.6/0.0	c_s= 0.1 c_i/c_o = 0.7/0.0	c_s= 0.0 c_i/c_o = 0.5/0.0

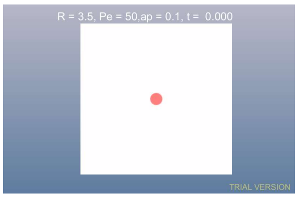


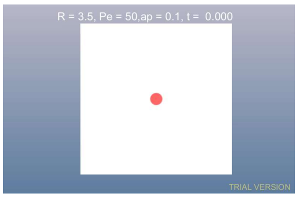


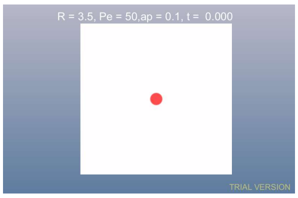


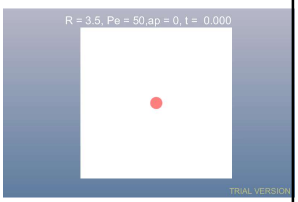


W/o per. W/o per. W/o per. W/o per.





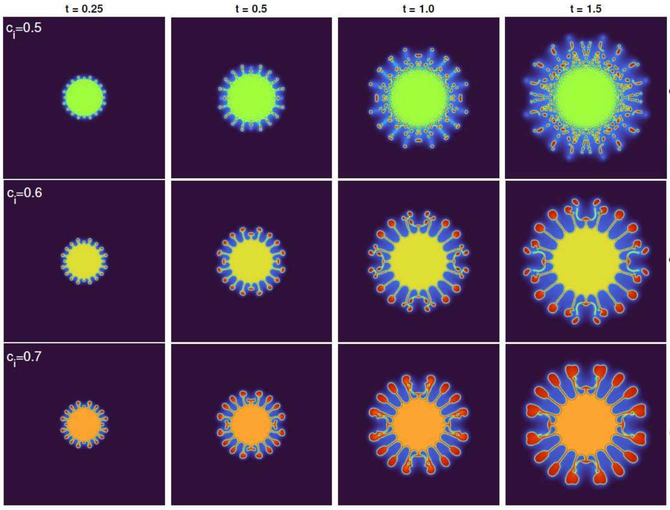




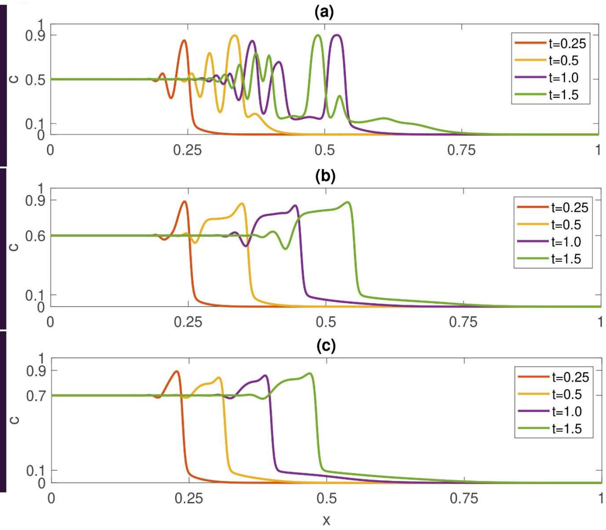
With per. With per. With per. With per.

Pattern formation : various c_i = 0.5, 0.6, 0.7 (θ⇩)

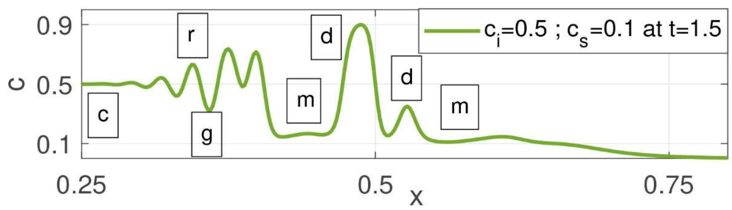
R = 3.5, Pe = 50, C = 10^{-5}, I = 12.5 & c_s = 0.1



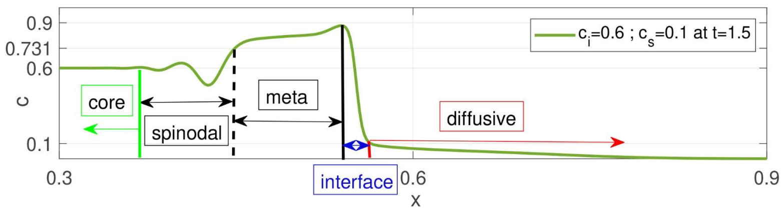
Concentration profile along centerline (y=0, 0<x<1)

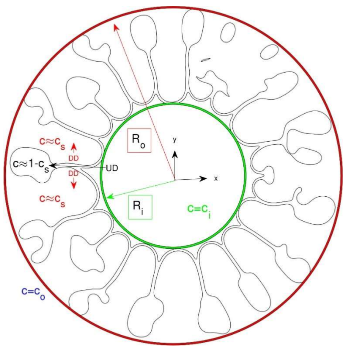


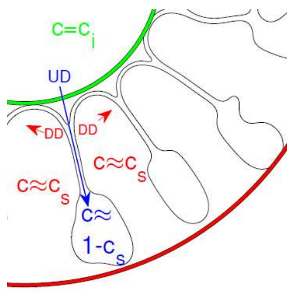
Concentration Distribution, c_s=0.1



Enlarged view of concentration profile along the centerline. The letters c, g, r, d, and m represent the core, groove, ridge, droplet, and mixing area, respectively.



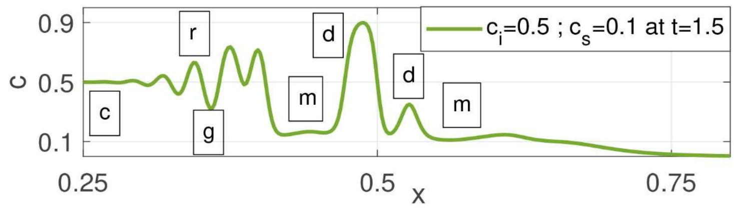




Uphill Diffusion (UD) & Downhill Diffusion (DD)

Anomalous patterns : Droplets and Lollipop Fingers

Sharp interface: c_s=0.1; 1-c_s=0.9



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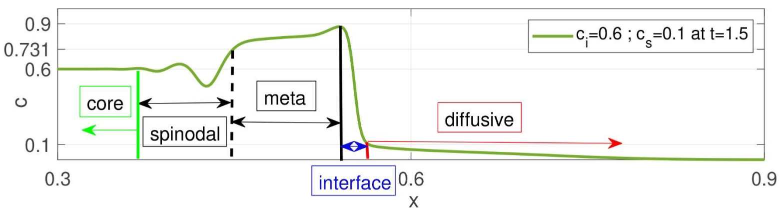
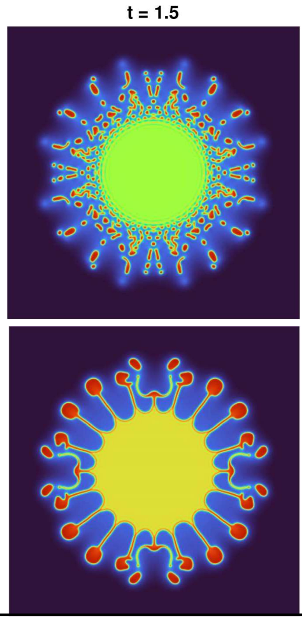
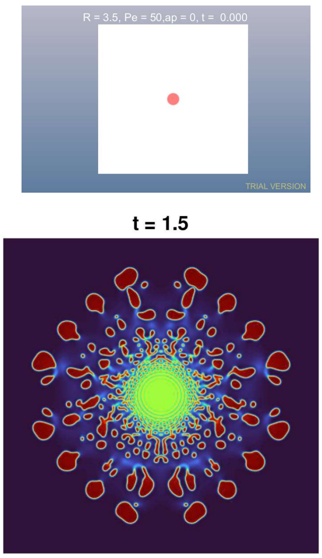




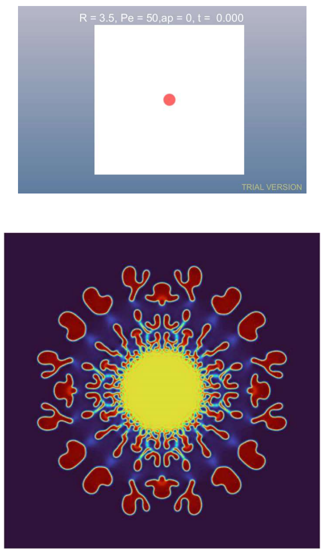
Spinodal: 0.269 < c < 0.731 ; Metastable: 0.731< c < 0.9

Pattern formation, c_s = 0.0, Strongest PS (Θ⇧)

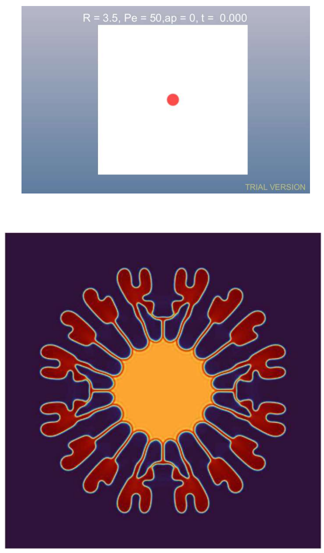
c_i/c_o = 0.5/0.0



c_i/c_o = 0.6/0.0

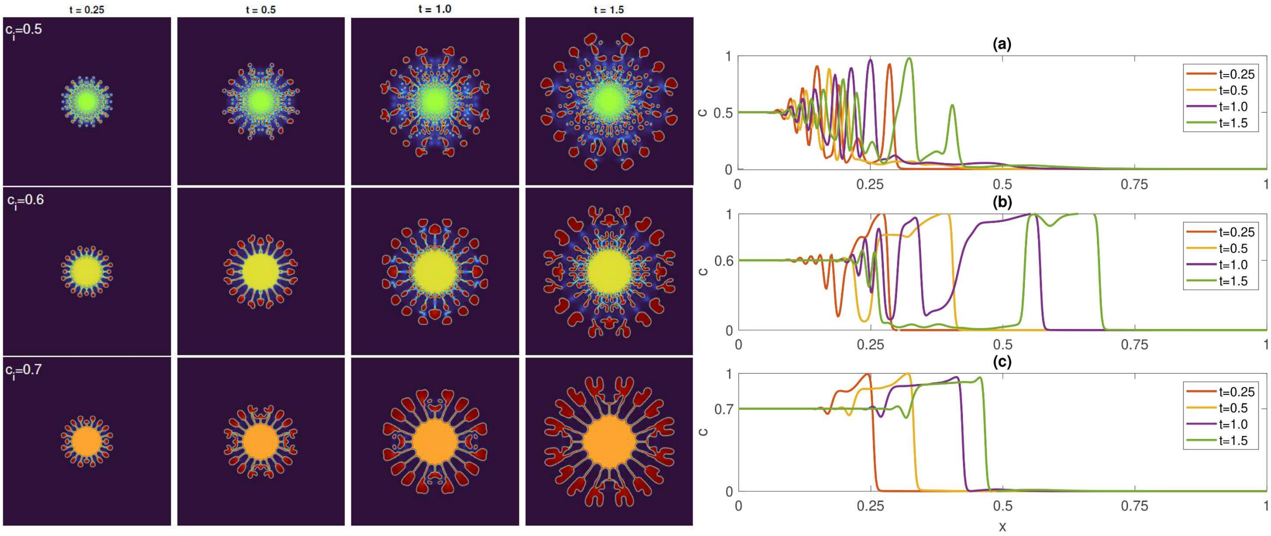


c_i/c_o = 0.7/0.0

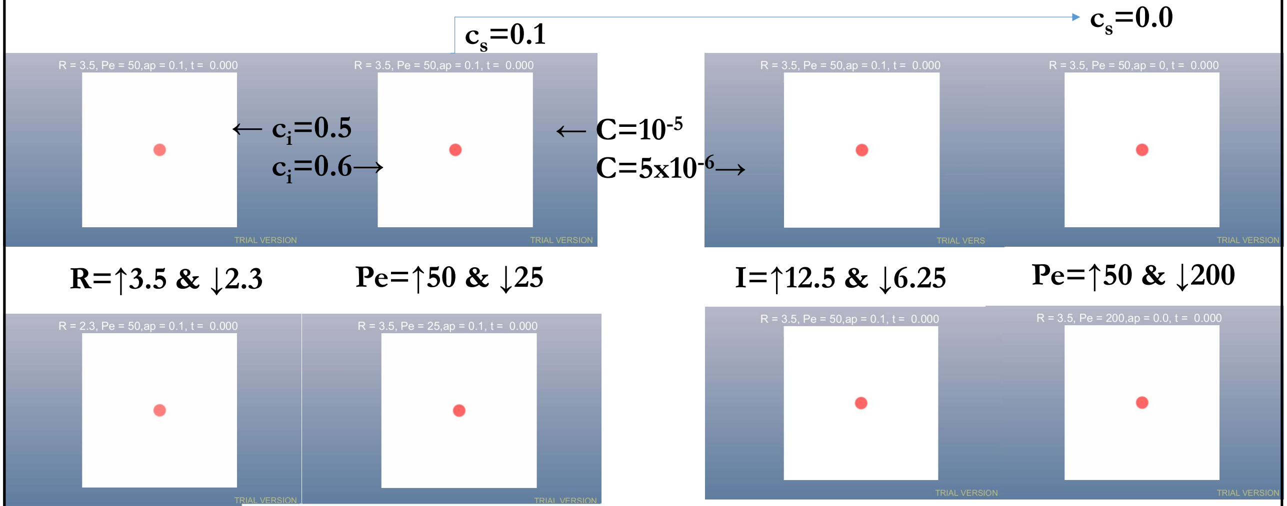


Pattern formation , c_s = 0.0, Strongest PS (θ⇧)

R = 3.5, Pe = 50, C = 10^{-5}, I = 12.5 & c_s = 0.   Concentration profile along centerline (y=0, 0<x<1)



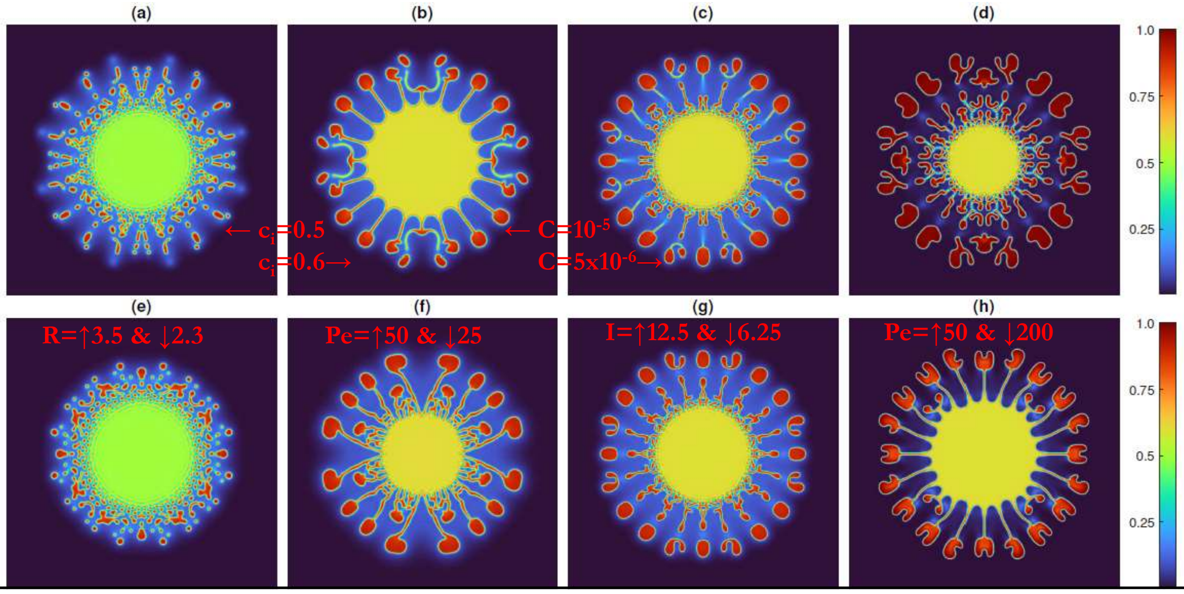
Parametric Analysis : Rv, Pe, C, I, c_s



$$Pe = \frac{4\rho R_c^2}{\alpha f^* t_c},\ R_v = ln(\frac{\eta_2}{\eta_1}),\ C = \frac{\epsilon}{4R_c^2 f^*},\ I = \frac{48\eta_1 R_c^2}{\rho b^2 f^* t_c}.$$

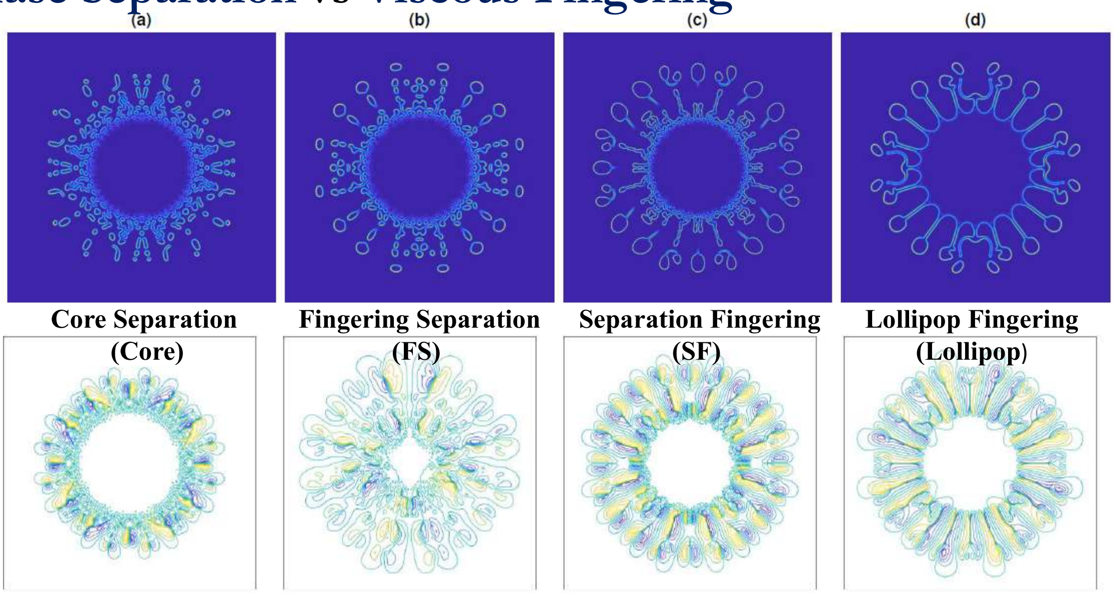
Parametric Analysis : R, Pe, C, I

(b & d) : $c_s = 0.1$ & $c_s = 0$

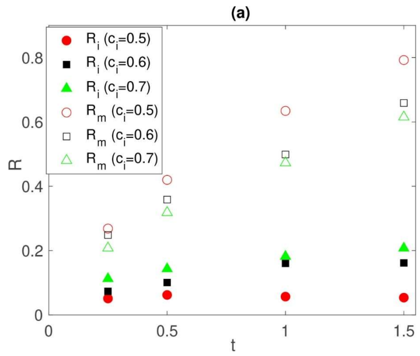


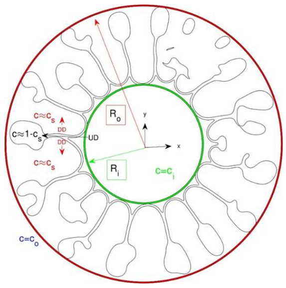
Four Categorized Patterns

Phase Separation vs Viscous Fingering

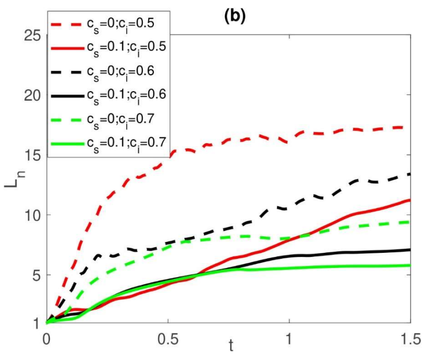


Quantitative Analysis





R_o : radius of circumscribed circle
R_i : radius of inscribed circle
R_m : Mixing radius, R_m=R_o-R_i

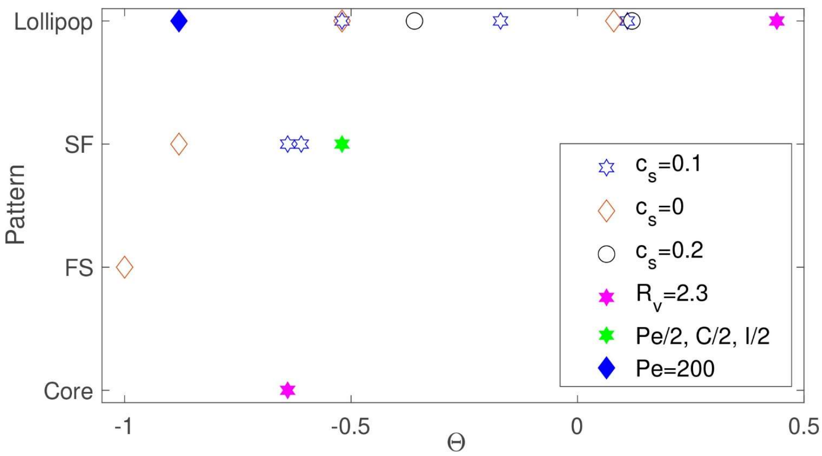


Normalized interfacial length
Ln=1 : circularly stable

(1) c_i ⇧ → R_m ⇧ (longer finger): higher viscosity contrast along core
(2) c_i ⇧ → R_i ⇩ : stronger core phase separation
(3) R_o ≅ constant : the same effective viscosity contrast along interface, i.e., c_s=0, c_o=0 and $\eta = \exp[R_v(1-c)]$
(4) Fingering separation (c_s = 0&c_i = 0.5) : R_i ⇩ vs t ⇧

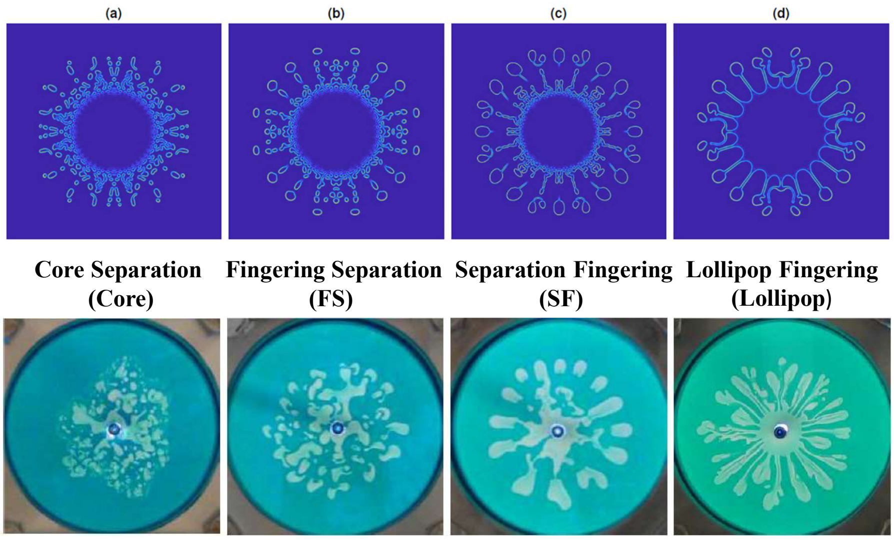
(1) PS ⇧ (c_i⇩ or c_s⇩) → Ln ⇧
(2) Early rapid growth due to PS : close growth rates with identical Θ, e.g., c_s = 0&c_i = 0.7 and c_s = 0.1&c_i = 0.6
(3) FS (c_s = 0&c_i = 0.5) : rapid PS at early time but level-off at later time
(4) SF (c_s = 0.1&c_i = 0.5) : continuous growth

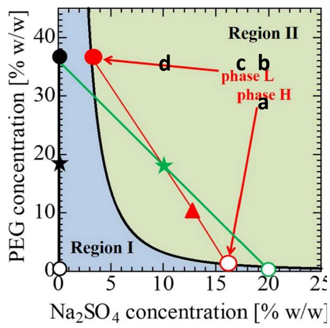
Phase Diagram of Pattern



$$Pe = \frac{4\rho R_c^2}{\alpha f^* t_c},\ R_v = ln(\frac{\eta 2}{\eta_1}),\ C = \frac{\epsilon}{4R_c^2 f^*},\ I = \frac{48\eta_1 R_c^2}{\rho b^2 f^* t_c}.$$

Qualitative comparison with experiments





Prominence of Phase Separation
(b) > (c) > (d)
Prominence of Viscous Fingering
(a) < (b) < (c) < (d)

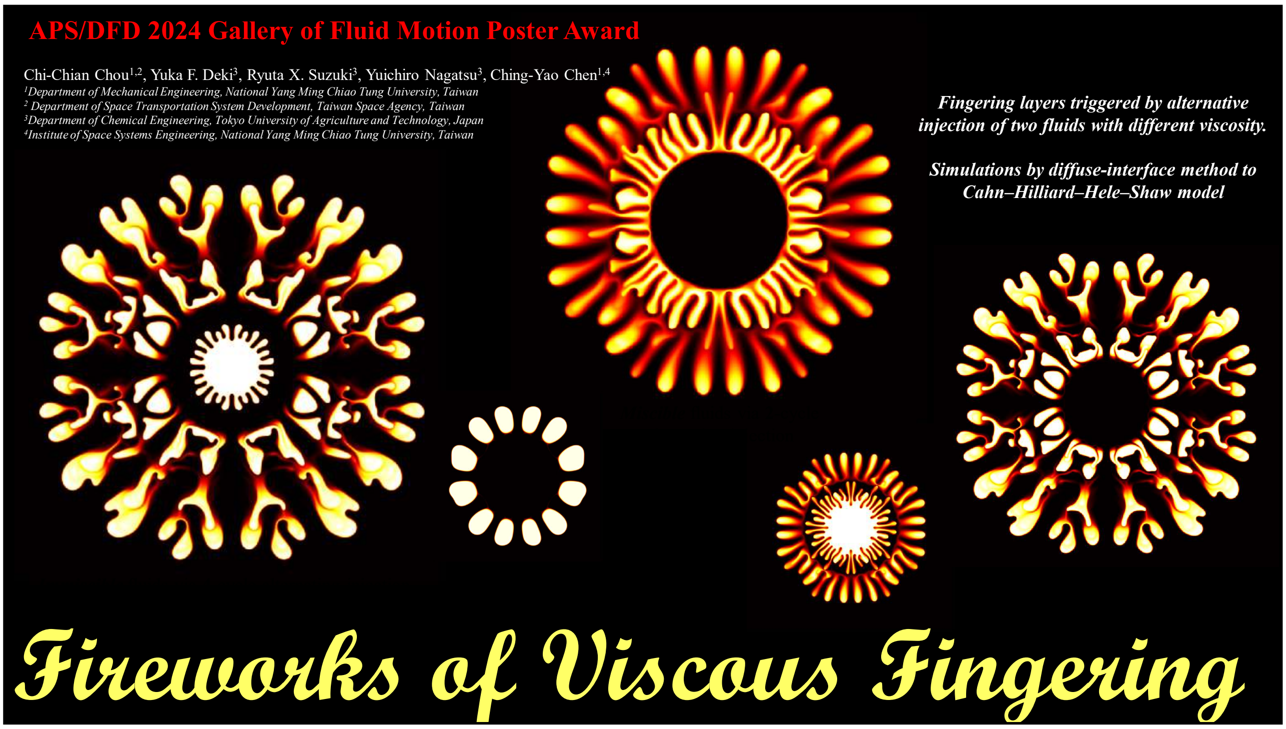
(a) Core⇨ (b) FS⇨ (c) SF⇨ (d) Lollipop

(a) 20wt % Na_2SO_4 to 30.0wt % PEG (R = 3.2) ; (b) 20wt % Na_2SO_4 to 36.5wt % PEG (R = 4.0) ;
(c) 17wt % Na_2SO_4 to 36.5wt % PEG (R = 4.2) ; (d) 10wt % Na_2SO_4 to 36.5wt % PEG (R = 4.5).

Summary

- Elucidating the effect of phase separation (PS) on viscous fingering (VF) numerically by a phase-field method in a partially miscible system
- Observing interesting patterns by the coupling effect of PS and VF
- Depending on the dominating mechanism, i.e., PS or VF, patterns categorized as (a) Core separation, (b) Fingering separation, (c) Separation fingering, (d) Lollipop Fingering.

Thank you!


APS/DFD 2024 Gallery of Fluid Motion Poster Award
Chi-Chian Chou[1,2], Yuka F. Deki[3], Ryuta X. Suzuki[3], Yuichiro Nagatsu[3], Ching-Yao Chen[1,4]
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[2] Department of Space Transportation System Development, Taiwan Space Agency, Taiwan
[3]Department of Chemical Engineering, Tokyo University of Agriculture and Technology, Japan
[4]Institute of Space Systems Engineering, National Yang Ming Chiao Tung University, Taiwan
Fingering layers triggered by alternative injection of two fluids with different viscosity.
Simulations by diffuse-interface method to Cahn–Hilliard–Hele–Shaw model
Fireworks of Viscous Fingering