Oral presentation | Multi-phase flow

Multi-phase flow-IV

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[4-D-02] A New Interface tracking method Under Multi-Material ALE Framework: The Intersecting Polygon Tracking Method

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Keywords: interface tracking, multi-material ALE, intersecting polygon

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A New Interface Tracking Method Under MMALE Framework: The Intersecting Polygon Tracking Method

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Outline

1. Introduction
2. Key Algorithms
3. Numerical Examples
4. Conclusions

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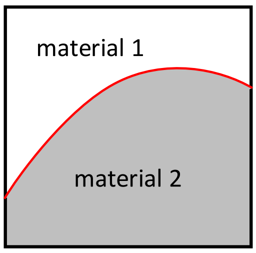
1、Introduction

- Modeling of moving interfaces is of great importance in immiscible multi-material flow simulations.
- In Lagrangian method, the material interfaces coincide with mesh lines so they are naturally captured.
- The Lagrangian mesh may be severely distorted due to local vorticity.
- Arbitrary Lagrangian-Eulerian (ALE) method[1]: Lagrangian + Rezoning + Remapping.
- Rezoning stage: Lagrangian mesh nodes are moved to more optimized position.
- Remapping stage: Lagrangian solution is conservatively transferred to rezoned mesh.

[1] Barlow A J , Maire P H , Rider W J ,et al.Arbitrary Lagrangian–Eulerian methods for modeling high-speed compressible multimaterial flows[J].Journal of Computational Physics, 2016.DOI:10.1016/j.jcp.2016.07.001.

1、Introduction

- In multi-material cases, the ALE method may generate mixed cells (two or more materials exist in one cell). The material interfaces become internal boundaries and the interface location must be calculated as part of the solution.



A mixed cell

- A family of numerical methods have been reported over last decades to represent and evolve the location of a sharp interface in multi-material simulations, such as Front Tracking, Level Set, Volume-of-Fluid and Moment-of-Fluid, etc.

1、Introduction: Front Tracking method

- **Front Tracking methods advect marker points on an initial interface with the flow so that a continuous, piecewise smooth interface approximation is known at each time step[1].**
- **Changes in topology can be taken into account, but this is nontrivial[2].**

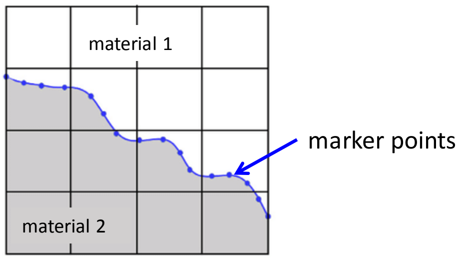


Illustration of front tracking method [3]

[1] Tryggvason, G.; Bunner, B.; Esmaeeli, A.; Juric, D.; Al-Rawahi, N.; Tauber, W.; Han, J.; Nas, S.; et. al..A Front-Tracking Method for the Computations of Multiphase Flow[J].Journal of Computational Physics,2001，169 (2) ：708-759.
[2] Wurigen Bo, Xingtao Liu, James Glimm, and Xiaolin Li.A Robust Front Tracking Method: Verification and Application to Simulation of the Primary Breakup of a Liquid Jet[J].SIAM Journal on Scientific Computing ,2011，33 (4) ：1505-1524.
[3] Luo, K. & Changxiao, Shao & Min, Chai & Fan, Jianren. (2019). Level set method for atomization and evaporation simulations. Progress in Energy and Combustion Science. 73. 65-94. 10.1016/j.pecs.2019.03.001.

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1、Introduction: Level Set method

- **In Level Set method [1-3], the interface is represented implicitly using a continuous level set function which is defined as signed distance function from mesh points to the interface.**
- **The interface location is identified by the contour of zero level set.**
- **Changes in topology are automatically taken into account.**
- **The interfaces are continuous and smooth, too.**
- **Level set method does not inherently conserve mass and requires artificial redistancing to ensure that the level set function remains a distance function.**

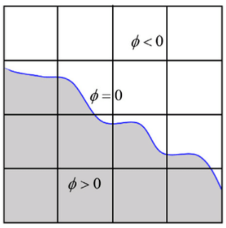

Illustration of level set method [4]

[1] F. Gibou, R. Fedkiw, S. Osher, A review of level-set methods and some recent applications, J. Comput. Phys. 353 (2017) 82–109.
[2] S. Osher, J.A. Sethian, Fronts propagating with curvature-dependent speed: algorithms based on Hamilton-Jacobi formulations, J. Comput. Phys. 79 (1)(1988) 12–49.
[3] M. Sussman, A level set approach for computing solutions to incompressible two-phase flow, J. Comput. Phys. 114 (1) (1994) 146–1
[4] Luo, K. & Changxiao, Shao & Min, Chai & Fan, Jianren. (2019). Level set method for atomization and evaporation simulations. Progress in Energy and Combustion Science. 73. 65-94. 10.1016/j.pecs.2019.03.001.

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1、Introduction: Volume-of-Fluid method

- In Volume-of-Fluid (VOF) method [1-3], the fractional volume of each material in a cell is tracked and, when needed, the interface is reconstructed from the volume fraction data.
- The most common reconstruction method is called Piecewise Linear Interface Calculation (PLIC), where the interface in a cell is approximated by a segment of a straight line.
- VOF methods are explicitly conservative and can simply manage dramatic topological changes in the interface structure without specialized algorithmic treatment.
- The interfaces are discontinuous, and some VOF schemes exhibit "numerical surface tension", of which one symptom is the division of thin filaments into a series of circular droplets.

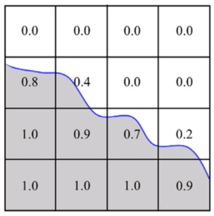

Illustration of VOF method [4]

[1] C.W. Hirt, B.D. Nichols, Volume of fluid (VOF) method for the dynamics of free boundaries, J. Comput. Phys. 39 (1) (1981) 201–225.
[2] D.L. Youngs, Time-dependent multi-material flow with large fluid distortion, in: Numerical Methods in Fluid Dynamics, Academic Press, 1982.
[3] W.J. Rider, D.B. Kothe, Reconstructing volume tracking, J. Comput. Phys. 141 (1997) 112–152.
[4] Luo, K. & Changxiao, Shao & Min, Chai & Fan, Jianren. (2019). Level set method for atomization and evaporation simulations. Progress in Energy and Combustion Science. 73. 65-94. 10.1016/j.pecs.2019.03.001.

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1、Introduction: Moment-of-Fluid method

- In Moment-of-Fluid (MOF) method[1], both material volume (zeroth moment) and material centroid (first moments) are tracked and utilized to reconstruct a piecewise linear interface.
- MOF method does not require any neighbor cell information for interface reconstruction. The numerical surface tension is drastically reduced.
- MOF method is highly sensitive to centroid location. Slight error in centroid location may change the interface orientation. So accurate centroid advection and exact matching of reconstructed centroid with reference centroid is critical to MOF method.
- The principal disadvantage of MOF method is computational cost: every interface reconstruction requires the solution of a nonlinear optimization problem. For a mixed cell containing more than 2 materials, the reconstruction procedure need to be applied recursively.

[1] Dyadechko V , Shashkov M .Reconstruction of multi-material interfaces from moment data [J].Journal of Computational Physics, 2008, 227(11):5361-5384.DOI:10.1016/j.jcp.2007.12.029.

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1、Introduction

- For PLIC methods, the local topology of the interface in a cell is quite limited (one half plane intersecting with the respective cell), making it difficult to capture subcell scale interface structures.
- MOF^2 method[1]: intersecting the cell with two half-planes. The configuration of interface is extended: half plane, corner, strip, bow-tie and their cell-complement. but still limited.
- Limitation: disconnected between neighbouring cells.

[1] Shashkov M , Kikinzon E .Moments-based interface reconstruction, remap and advection[J].J. Comput. Phys. 2022, 479:111998.

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1、Introduction

Main considerations when constructing new interface tracking method:

- mass conservation
- subcell scale structure resolving
- continuity

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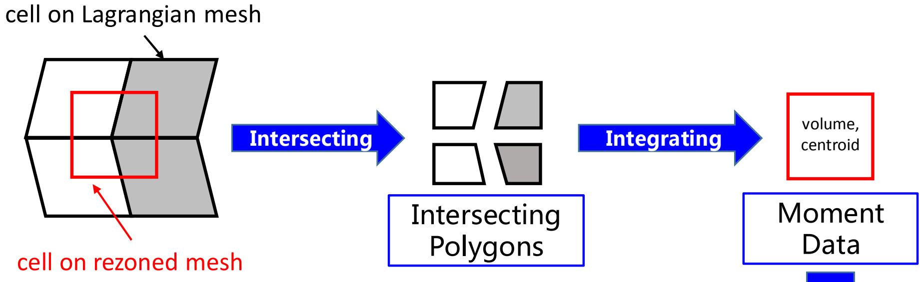
1、Introduction

☐ Polygon intersecting based remapping:



This 'Integrating-Reconstructing' process may cause information loss.

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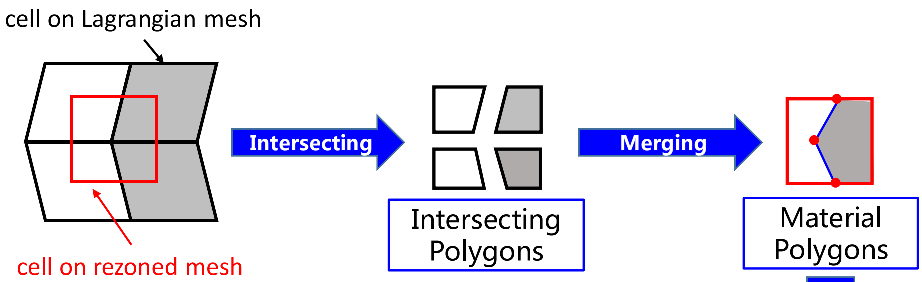
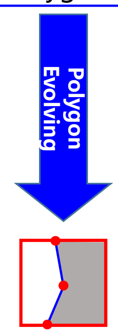
1、Introduction

☐ What if we keep these intersecting polygons?



- ✓ **Intersecting polygons are merged to construct material polygons;**
- ✓ **Material polygons are evolved in Lagrangian step;**
- ✓ **Material Interfaces are exactly the common faces of adjacent material polygons.**



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2、Key algorithms

- ☐Computational graphics algorithm;
- ☐Polygon evolution algorithm;
- ☐Interface optimization algorithm.

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2.1、Computational Graphics algorithm

We use GPC library to fulfill the following tasks:

- ✓ polygon intersection operation.
- ✓ polygon union operation.

https://github.com/lguez/GPC

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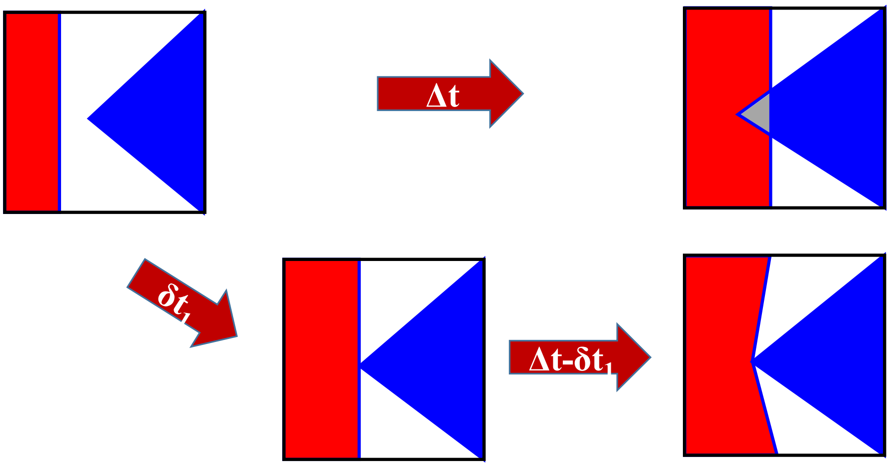
2.2、Polygon Evolution Algorithm

Vertex velocity: intropolation of cell node velocity

1. topology change:



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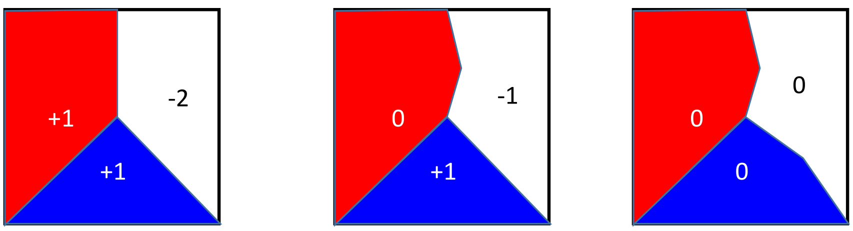
2.2、Polygon Evolution Algorithm

Vertex velocity: intropolation of cell node velocity

2. conservation for incompressible flows: volume exchange



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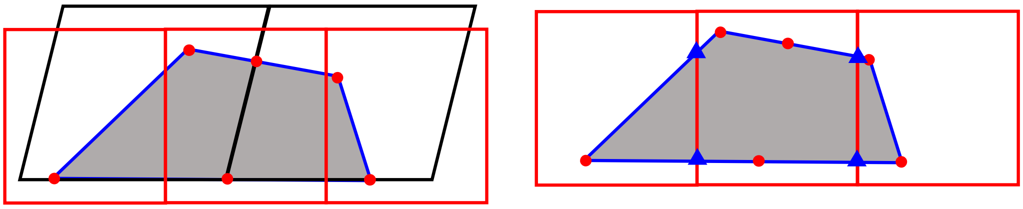
2.3、Interface Optimization Algorithm

- In remapping stage, new vertices are inserted into material polygons.



- Too much details, increasing computation cost without improving accuracy.
- Optimizing rules: volume unchanged, interface continuity.

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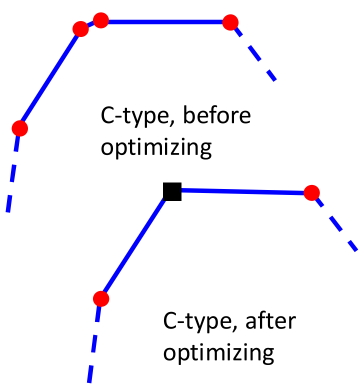
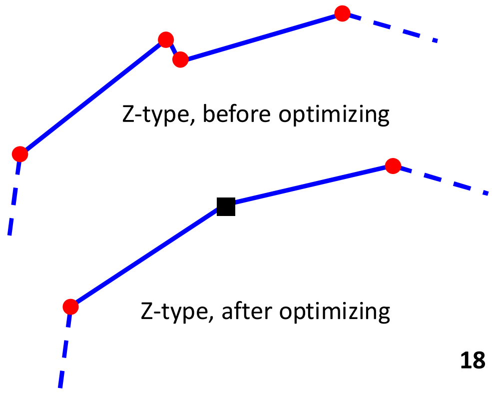
2.3、Interface Optimization Algorithm

- There are two typical situations where we can eliminate one polygon vertex: C-type and Z-type.





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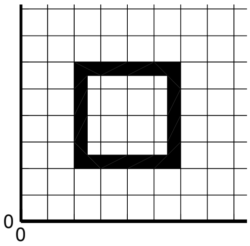
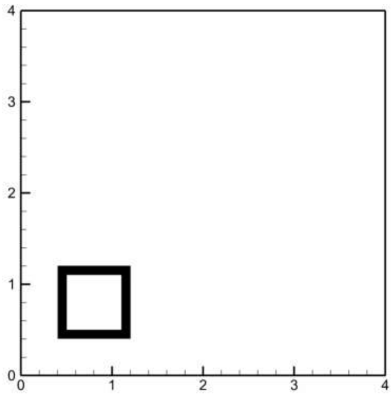
3、Numerical Examples

Hollow square translating

- computation domain: 4×4
- mesh resolution: 20×20
- mesh size: 0.2
- square side length: inner 0.6, outer 0.8
- initial center location: (0.8,0.8)
- velocity field (u,v) = (2,1)
- final time: t = 1.25



Initial mesh and material configuration



- ✓ Although the material width is smaller the mesh size, the interfaces are still integrated and connected.
- ✓ The sharp corners are well maintained.

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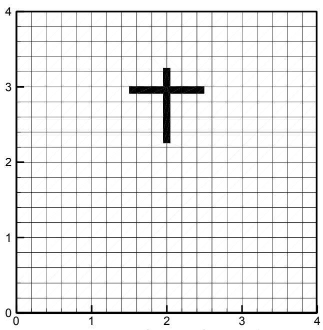
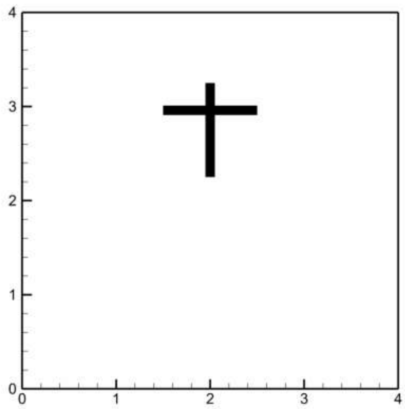
3、 Numerical Examples

Cross-shape rotating

- computational domain: 4×4
- grid resolution: 20×20
- velocity field: $\begin{cases} u = -0.5(y-2.0) \\ v = \quad 0.5(x-2.0) \end{cases}$
- time duration: t = 4π



Initial mesh and material configuration



- ✓ Although the material width is smaller the mesh size, the interfaces are still integrated and connected.
- ✓ The sharp corners are well maintained.

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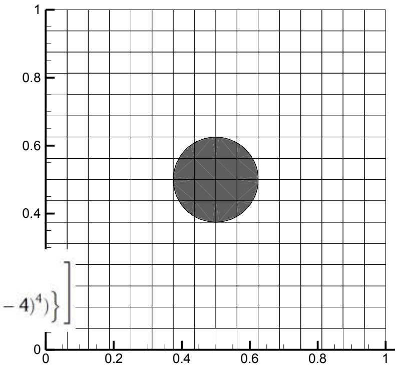
3、 Numerical Examples

Droplet flow[1]

- computation domain: 1×1
- mesh resolution: 16×16
- disk radius: 0.125
- center: (0.5,0.5)
- velocity field: $\mathbf{v} = \begin{bmatrix} \frac{1}{8}(8x-4) \\ \frac{1}{8}\left\{-(8y-4) - 4 - (1-(8x-4)^2-(8x-4)^4)\right\} \end{bmatrix}$
- final time: t=0.75



[1] Ahn H T , Shashkov M .Adaptive moment-of-fluid method[J].Journal of Computational Physics, 2009, 228(8):2792–2821.

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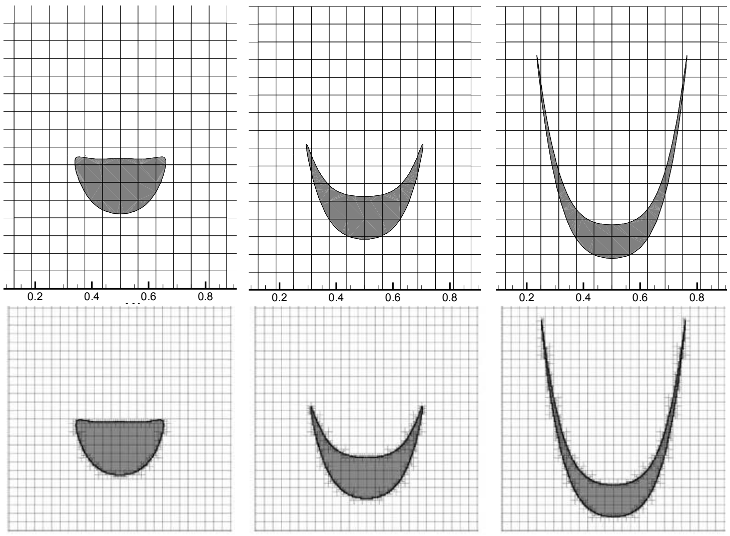
3、 Numerical Examples

Droplet flow



time=0.25 time=0.50 time=0.75

The present method, mesh resolution: 16×16

AMR-MOF[1], effective mesh resolution: 1024×1024

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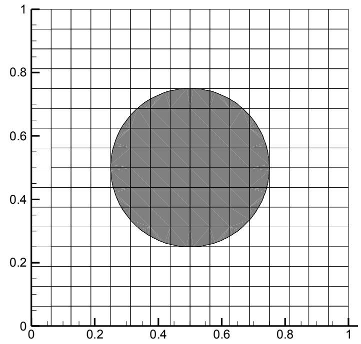
[1] Ahn H T , Shashkov M .Adaptive moment-of-fluid method[J].Journal of Computational Physics, 2009, 228(8):2792–2821.

3、 Numerical Examples

S-shape flow[1]

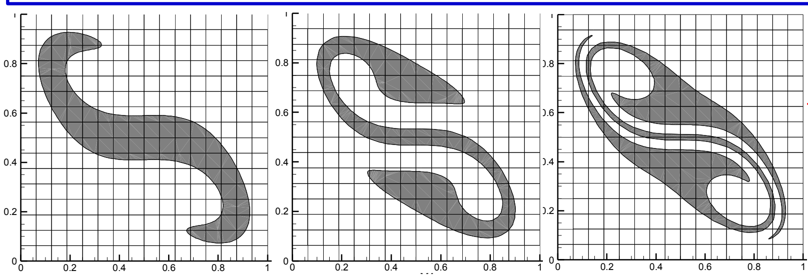
- computation domain: 1×1
- mesh resolution: 16×16
- disk radius: 0.25
- center: (0.5,0.5)
- velocity field: $\mathbf{v} = \begin{bmatrix} \frac{1}{4}\left\{(4x-2)+(4y-2)^3\right\} \\ \frac{-1}{4}\left\{(4y-2)+(4x-2)^3\right\} \end{bmatrix}$
- final time: t=3.0



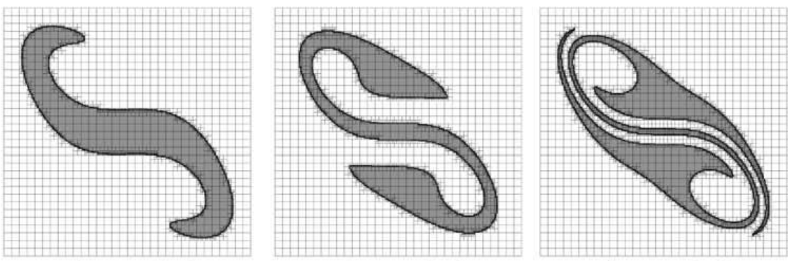
[1] Ahn H T , Shashkov M .Adaptive moment-of-fluid method[J].Journal of Computational Physics, 2009, 228(8):2792–2821.

3、 Numerical Examples

□ S-shape flow



The present method, mesh resolution: 16×16

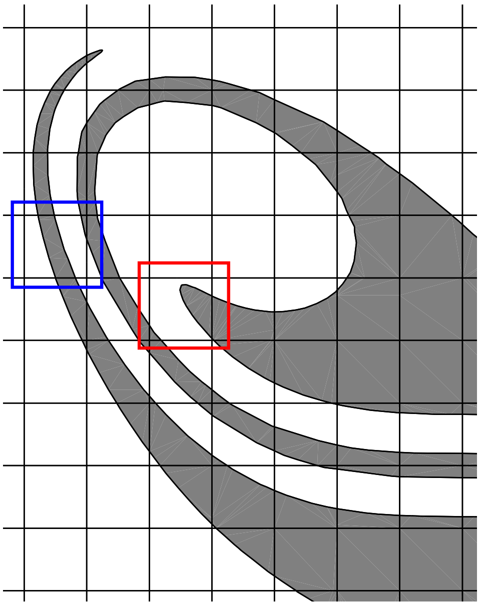


AMR-MOF[1], effective mesh resolution: 512×512

time=1.0 time=2.0 time=3.0

[1] Ahn H T , Shashkov M .Adaptive moment-of-fluid method[J].Journal of Computational Physics, 2009, 228(8):2792–2821.

3、 Numerical Examples



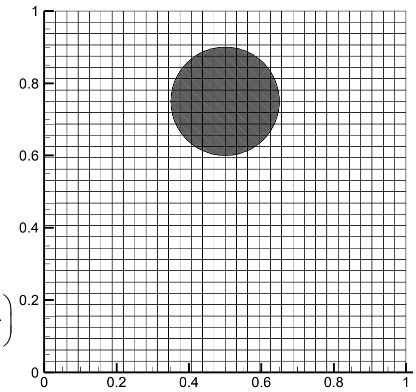
- blue : thin filament in cells
- red : complicated interface structure

3、 Numerical Examples

Reversible vortex, long period[1]

- computation domain: 1×1
- mesh resolution: 32×32
- disk radius: 0.15
- center: (0.5,0.75)
- velocity field:
- period: T=8
- final time: t=8

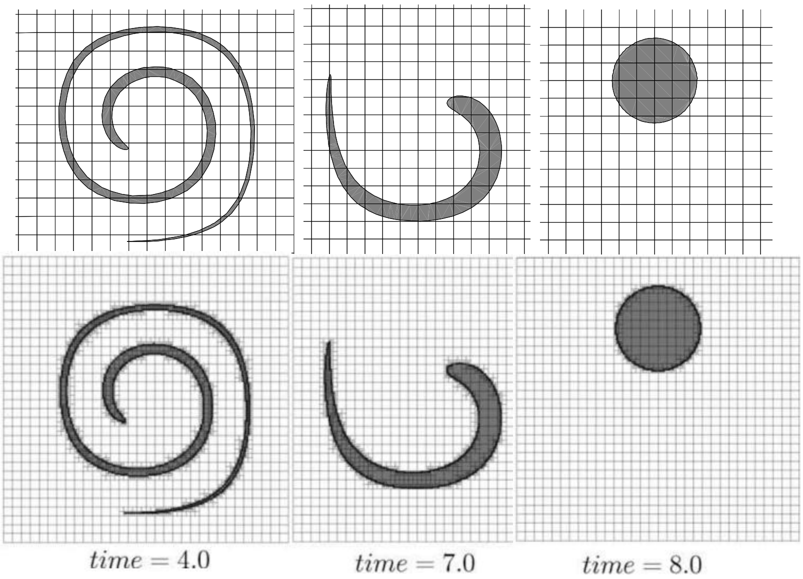
$$\begin{cases} u = \sin^2(\pi x)\sin(2\pi y)\cos\left(\frac{\pi t}{T}\right) \\ v = -\sin^2(\pi y)\sin(2\pi x)\cos\left(\frac{\pi t}{T}\right) \end{cases}$$



[1] Ahn H T , Shashkov M .Adaptive moment-of-fluid method[J].Journal of Computational Physics, 2009, 228(8):2792–2821.

3、 Numerical Examples

□ Reversible vortex



the present method, mesh resolution: 32×32

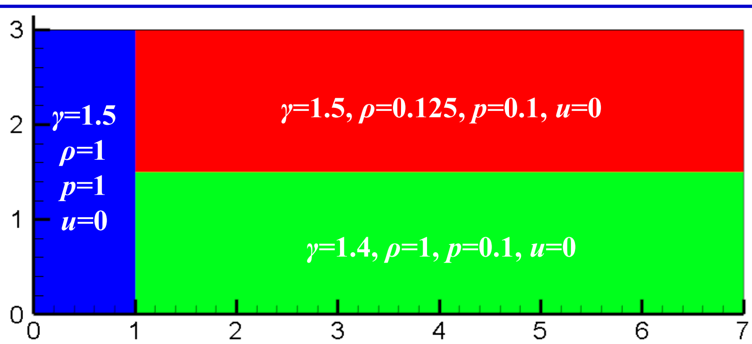
AMR-MOF[1], effective mesh resolution: 512×512

[1] Ahn H T , Shashkov M .Adaptive moment-of-fluid method[J].Journal of Computational Physics, 2009, 228(8):2792–2821.

3、 Numerical Examples

Triple point problem[1]

- **computation domain: 7×3**
- **mesh resolution: 210×90**
- **final time: t=5**

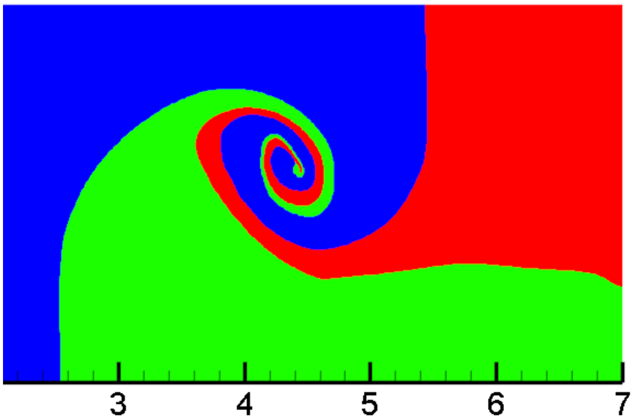


Initial condtion

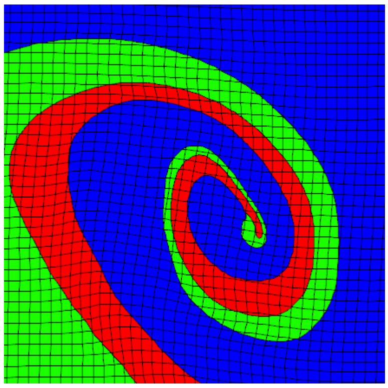
[1] Kucharik M , Garimella R V , Schofield S P ,et al.A comparative study of interface reconstruction methods for multi-material ALE simulations[J].Journal of Computational Physics, 2010, 229(7):2432-2452.

3、 Numerical Examples

□ Triple point problem



Material distribution at final time



Material distribution in the vicinity of the triple pont

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4、 Conclusions

We developed a new interface tracking method under MMALE framework. The main features of this method are as follows:

- ✓ **Intersecting polygons generated in remapping stage are used to track material interfaces;**
- ✓ **Subcell scale structures of material interface can be well captured;**
- ✓ **Material interfaces are continuous across cells;**
- ✓ **Computational cost will not increase dramatically with the increase of number of materials included in a cell.**

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Thank you for your attention!

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