Characteristics of Linearly-Forced Scalar Mixing in Homogeneous, Isotropic Turbulence

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Abstract: To realize the potential of direct numerical simulation (DNS) in turbulent mixing studies, it is critical to develop a scalar forcing methodology which preserves the inherent isotropy and homogeneity of the scales of turbulence. In this work, a linear forcing scheme is proposed that preserves these characteristics and it is applied to the simulation of unity and high Schmidt number turbulent flows.

Keywords: Direct numerical simulation, scalar forcing, turbulent mixing.

1 Introduction

The transport and mixing of passive scalars in turbulent flows remains an active area of research. To maximize the Reynolds and Schmidt numbers accessible via DNS, the velocity and scalar fields are often forced. The velocity field can be forced either spectrally at large wavelengths or linearly using the velocity fluctuations themselves [1]. An advantage to a linear forcing scheme is its simplicity and applicability to non-spectral codes. Further, the results obtained from linearly-forced velocity fields are comparable to those forced spectrally, which are illustrated in Fig. 1a. Forcing the scalar field, however, is slightly more complicated. Two existing forcing schemes have gained broad acceptance. The first involves imposing a mean scalar gradient,

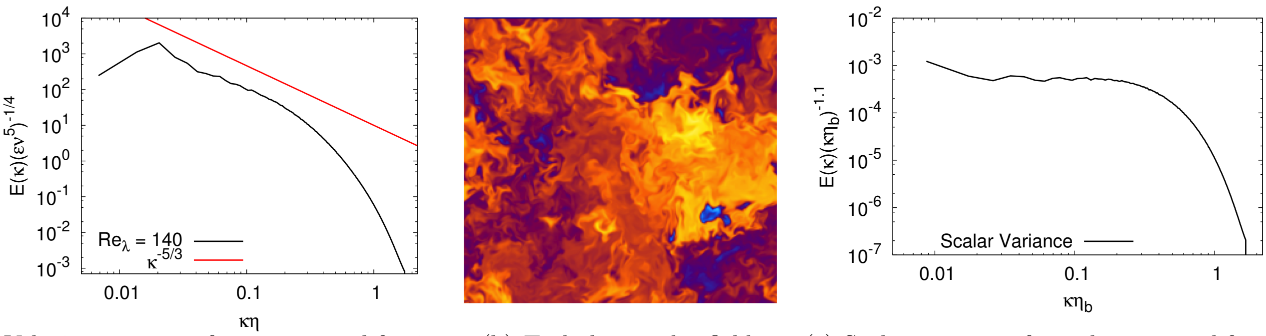
$$\frac{\partial Z}{\partial t} + \mathbf{u} \cdot \nabla Z = D \nabla^2 Z - \mathbf{G} \cdot \mathbf{u}, \tag{1}$$

where G represents the mean scalar gradient applied to the advection-diffusion equation for the scalar, Z [2]. Unfortunately, this anisotropic forcing method induces non-homogeneity in the resulting scalar field. The second method employs a solenoidal random forcing, f_z,

$$\frac{\partial Z}{\partial t} + \mathbf{u} \cdot \nabla Z = D \nabla^2 Z + \mathbf{f}_z. \tag{2}$$

This approach has been used extensively to study the behavior of scalar mixing when the Schmidt number is close to unity [3].

The focus of this work is 1) to develop a new scalar forcing scheme motivated by the linear velocity forcing of Lundgren [1], and 2) to validate it for a broad range of Schmidt numbers, including Sc $\approx$ 1 and Sc » 1. Focus is placed on the homogeneity and isotropy of the resulting



(a) Velocity spectrum from a spectral forcing. (b) Turbulent scalar field. (c) Scalar spectrum from the proposed forcing.

Figure 1: Velocity and scalar field behavior in homogeneous, isotropic turbulence.

scalar field, which is depicted in Fig. 1b. The resulting compensated scalar spectrum is provided in Fig. 1c.

2 New Scalar Forcing

This work proposes a linear scalar forcing as follows,

$$\frac{\partial Z}{\partial t} + \mathbf{u} \cdot \nabla Z = D \nabla^2 Z + \left[\frac{1}{\tau_I} \left(\frac{1}{\sqrt{Z_v}} - 1 \right) + \frac{1}{2} \frac{\chi}{Z_v} \right] Z, \tag{3}$$

where τ_I, Z_v, and χ are the relaxation time-scale, scalar variance, and dissipation rate in the entire domain, respectively. The proposed forcing function in the advection-diffusion equation is composed of a relaxation term (left) and a production term (right). The relaxation term allows the field to decay towards a specified variance, which was here set to unity. The production term is designed to balance exactly any losses due to dissipation. This forcing scheme has the advantage of being truly isotropic and statistically stationary, as the scalar field will evolve towards a constant variance. In addition, the forcing preserves the linearity of the original, unforced scalar transport equation.

The validation of this new scheme is conducted in two parts. First, the effect of the forcing on the symmetry, isotropy, and homogeneity of the turbulent scalar field is examined by evaluating the scalar energy spectrum and the probability density function of the scalar flux. Then, the efficacy of this approach is investigated by simulating both close to unity and high Schmidt number turbulent flows. Finally, this work includes a comparison between the proposed scalar variance forcing method and the widely-used mean scalar gradient forcing scheme.

3 Future Work

While the present study focuses on constant density flows, DNS will be performed to elucidate the effects of the scalar forcing in flows with non-constant density. This is key to understanding the evolution of instabilities in turbulent buoyant and reacting flows.

References

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