Simulation of Decaying Two-Dimensional Turbulence Using Kinetically Reduced Local Navier-Stokes Equations

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Abstract: Kinetically reduced local Navier-Stokes (KRLNS) equations are applied for simulation of decaying two-dimensional (2-D) homogeneous isotropic turbulence in order to demonstrate its capability to capture the correct transient behavior. The numerical results obtained by the KRLNS equations are compared with those obtained by the artificial compressibility method (ACM), the lattice Boltzmann method (LBM) and the pseudo-spectral method (PCM). The divergence as a function of time in the KRLNS method is compared with that of the ACM. It is confirmed that the KRLNS method can capture the correct transient behavior without use of sub-iterations due to a smoothing effect.

Keywords: Unsteady Incompressible Viscous Flow, Kinetically Reduced Local N-S Equations, Artificial compressibility Method, Lattice Boltzmann Method.

1 Introduction

Recently, an alternative thermodynamic description of incompressible fluid flows was suggested in the form of kinetically reduced local Navier-Stokes (KRLNS) equations and the capture of the correct time dynamics was studied [1]. In this paper, in order to investigate the capability to capture the correct transient behavior of the KRLNS equations, numerical simulations of decaying two-dimensional turbulence are carried out and the results are compared with the solutions obtained by the artificial compressibility method (ACM), the lattice Boltzmann method (LBM) and the pseudo-spectral method (PSM).

2 Kinetically Reduced Local Navier-Stokes Equations and Numerical Method

The classical incompressible Navier-Stokes equations consist of the equation for the velocity

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} + \nabla p = -\frac{1}{Re} \nabla \cdot \mathbf{u},$$

and the incompressibility constraint

$$\nabla \cdot \mathbf{u} = 0,$$

where \( \mathbf{u} \) is the fluid velocity, \( p \) is the pressure and Re is the Reynolds number. In the ACM, the time derivative of the pressure is introduced into the continuity equation for coupling between the pressure and the velocity. The continuity equation (2) is then written as

$$\frac{1}{\delta} \partial_t p = -\partial_a u_a,$$

where \( \delta \) is the artificial compressibility parameter, \( t \) is an auxiliary variable that can be related to the physical time. To satisfy the continuity equation (2), the sub-iterations at each time step is mandatory.

In the form of KRLNS equations, the pressure equation (3) is replaced by

$$\partial_t G = -\frac{1}{Ma^2} \partial_a u_a + \frac{1}{Re} \partial_a \partial_a G, \quad p = G + \frac{u^2}{2}.$$
where $Ma$ is the Mach number, $G$ is the grand potential. Retaining the term $1/\text{Re} \partial_p \rho \partial_p G$ is crucial for capturing the correct transient behavior without sub-iterations. In the numerical method for solving the KRLNS equations, a central difference scheme is used for the spatial discretization and 4 stage Runge-Kutta method is used in the time integration.

3 Numerical Results

The vorticity contour plot obtained by the KRLNS equations for decaying 2-D turbulence at $\text{Re} = 10000$ on a uniform $256 \times 256$ Cartesian grid is shown in Figure 1(a), while Figure 1(b) shows that obtained by the LBM. The plot obtained by the PSM, which is a standard approach to the direct simulation of turbulence, is shown in Figure 1(c). The agreement among the three simulations is excellent. The divergence as a function of time for the KRLNS equations is shown in Figure 2. The adjustable parameters for the KRLNS equations is $Ma = 0.02$, $\Delta t = 1 \times 10^{-4}$. It is confirmed that the KRLNS approach can keep the divergence fluctuation at the $10^{-2}$ level, while that in the ACM is much larger.

References