Deterministic Solver for Steady State Problems of Gases of Arbitrary Statistics Based on the Semiclassical Boltzmann - BGK Equation.

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Abstract: Following the developed explicit method by [3], the implicit method derived using the lower-upper (LU) factorization for solving the semiclassical Boltzmann-BGK equation is presented. Boundary value problems of steady and unsteady rarefied gas flows of Fermi-Dirac, Bose-Einstein and Maxwell-Boltzmann statistics are considered.
Keywords: Semiclassical Boltzmann-BGK Equation, Discrete Ordinate Method, Particle Statistics, Implicit Schemes with LU Factorization.

1 Introduction

An algorithm for solving the semiclassical Boltzmann equation based on Bhatnagar-Gross-Krook [1] relaxation time approximation for gases of arbitrary statistics is presented. The discrete ordinate method is first applied to render the Boltzmann equation into hyperbolic conservation laws with source terms. The TVD [2] spatial operator is employed for fluxes calculation. In the abstract the problem of two-dimensional steady semiclassical gas flow problem is tested using the implicit scheme with LU factorization.

2 Semiclassical Boltzmann-BGK Equation

The governing equation to solve is the semiclassical Boltzmann-BGK equation with relaxation time approximation which reads

$$\left(\frac{\partial}{\partial t} + \frac{\mathbf{p}}{m} \cdot \nabla_x - \nabla U(\mathbf{x}, t) \cdot \nabla_{\mathbf{p}}\right) f\left(\mathbf{p}, \mathbf{x}, t\right) = -\frac{f - f^{(0)}}{\tau} \tag{1}$$

The equilibrium distribution function for general statistics can be expressed as

$$f^{(0)}\left(\mathbf{p}, \mathbf{x}, t\right) = \frac{1}{z^{-1}\, exp\left\{\left[\mathbf{p} - m\,\mathbf{u}(\mathbf{x}, t)\right]^2 / 2mk_B T(\mathbf{x}, t)\right\} + \theta} \tag{2}$$

where $\mathbf{u}(\vec{x}, t)$ is the mean velocity, $T(\mathbf{x}, t)$ is temperature, k_B is the Boltzmann constant and $z\left(\mathbf{x}, t\right) = exp\left(\mu(\mathbf{x}, t)/k_B T(\mathbf{x}, t)\right)$ is the fugacity, where μ is the chemical potential. $\theta = 1$

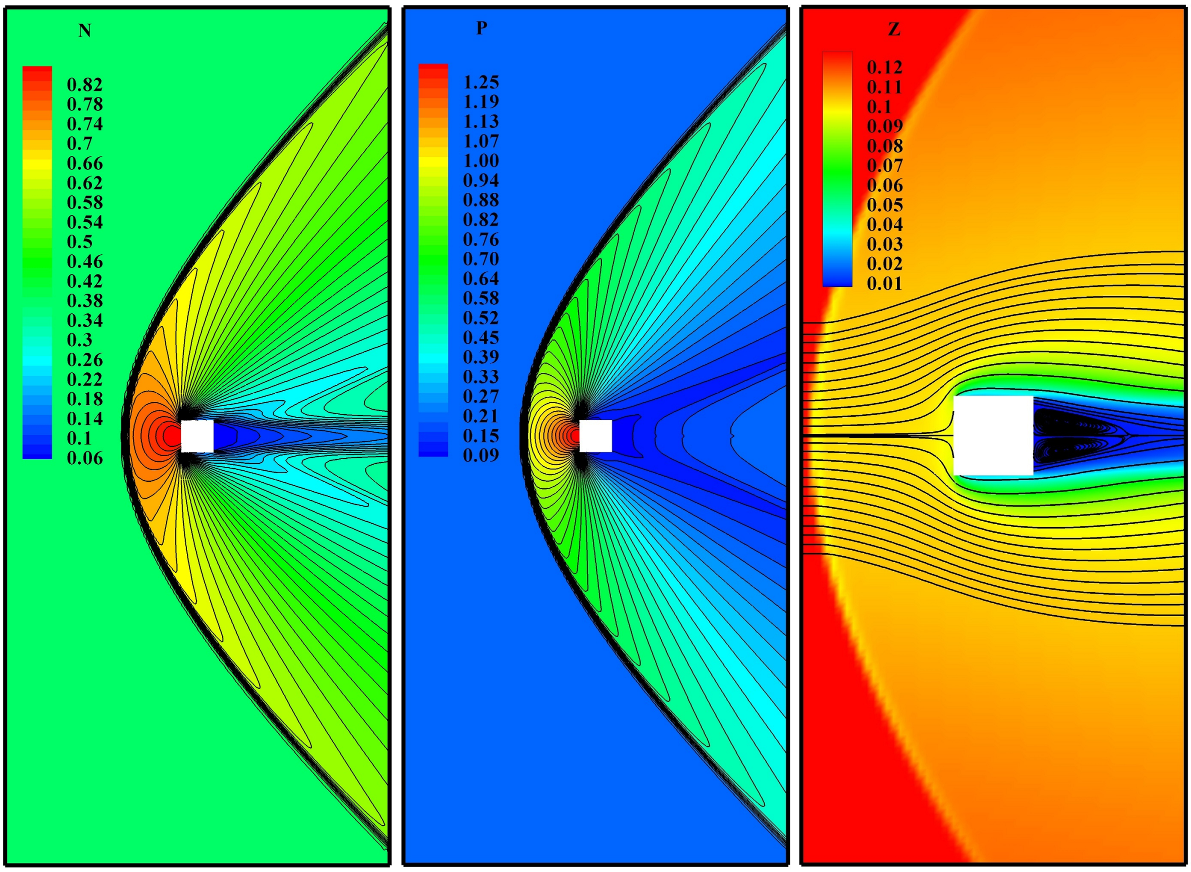

Figure 1: (left) Density, (center) Pressure and (right) Fugacity Contours and Streamline of Fermi-Dirac Gas near Continuum Regime with $\tau = 0.0005$

denotes the Fermi-Dirac statistics, $\theta = -1$, the Bose-Einstein statistics and $\theta = 0$ denotes the Maxwell-Boltzmann statistics.

3 Preliminary Results and Future Works

Fig. 1 shows the preliminary result of Fermi-Dirac gas flow with free stream Mach number 2 over a square cylinder upon convergence. Implicit schemes with LU Decomposition is applied on a uniform Cartesian grid with $\Delta x = \Delta y = 0.09$. Small relaxation time $\tau = 0.0005$ is used to model a flow near continuum regime. Specular boundary condition is assumed and multi valued points at wall edges are used. 20×20 points Gauss-Hermite quadrature is applied. The contours of density, pressure and fugacity along with the steady bow shocks at steady state are depicted. Convergence is assumed to occur when the L_2 norm of the residual is reduced less than 10^-7. More rigorous testings for all the other statistics covering various relaxation times will be reported in the final paper.

References

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