Study on the Computational Efficiency of Three Dimensional Euler Equations by Multi-Resolution Analysis

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Abstract: In this study, efficient computational algorithm for three dimensional Euler equations is developed based on the multi-resolution analysis. A proper thresholding technique is applied to preserve the order of the accuracy of the original solver. In order to assess the efficiency improvement of proposed algorithm, the method is applied to the computation of flow field around ONERA-M6 wing in transonic regime. The computation time is significantly reduced by the appropriate setup of the computational domain where the actual flux solver is applied.

Keywords: Multi-resolution analysis, Computational Efficiency, Three Dimensional Euler Equations.

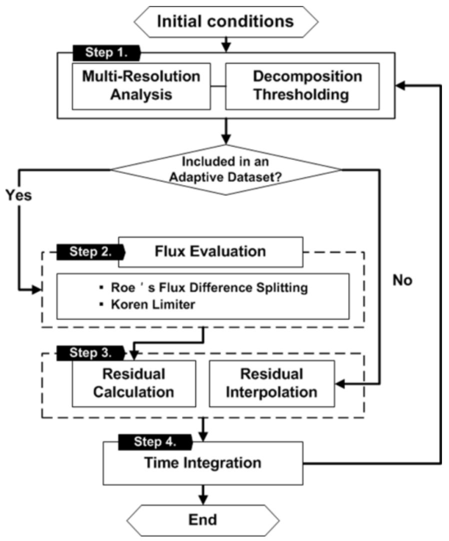
1 Introduction

As the massive computation becomes more common, for instance, for the accurate analysis of complicated flows related with transonic/supersonic flow phenomena, the efficiency of numerical simulations has become more important. The efficiency issue can be attacked by numerical algorithms via multi-resolution analysis. Harten develped the multi-resolution scheme for hyperbolic PDEs [1]. Holmström proposed the algorithm that uses the interpolating wavelet transformation to organize an adaptive dataset [2]. Kang et. al. presented a modified multi-resolution analysis and enhanced the computational efficiency of the solution [3].

The objective of this study is the analysis of the multi-resolution method in the three dimensional Euler equations. The multiresolution algorithm is applied to the computation of transonic flow field aroud the ONERA-M6 wing in order to assess the actual efficiency of the algorithms.

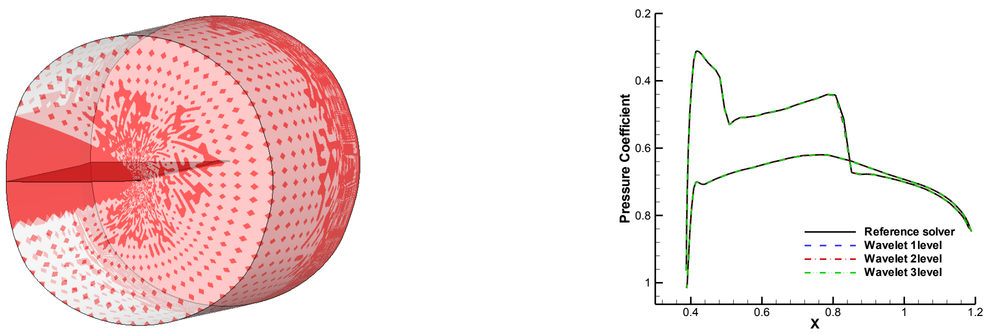
2 Problem Statement

The overall procedure of the proposed algorithm is shown as Figure 1. It consists of 4 steps: multi-resolution analysis for three dimensional decomposition process with thresholding, flux evaluation, residual calculation / residual interpolation and time integration. In the multi-resolution analysis, the less important informations (details) in the dataset are thresholded that actually allows the efficient information process. Therefore, the careful design of the thresholding procedure is highly important to preserve the order of the accuracy of the original solution, that will be shown in the full paper. For assessment of the algorithm, it is applied to the computation of flow field around the ONERA-M6 wing in transonic regime (M=0.8395, AOA=3.06°). The informations need to be updated only at the grid points where flow rapidly changes such as near the wing and downstream as shown in Figure 2 (a). Figure 2 (b) shows pressure coefficient distributions at 50% of the wing span with shock location information. The solution algorithm with and without the application of the wavelets, produce nearly the same pattern of solution. Table 1 shows the effect of the computation speedup with the proposed algorithm which is 3 times faster than conventional CFD solver independent of the resolution level which is different from 2D analysis. L_2 error norm between conventional solver and the proposed multi-resolution algorithm remain only about $O(10^{-7})$.



- **Step 1:** The multi-resolution analysis is performed; by the decomposition and thresholding, computational domain is adapted to follow the local features of the solution such as shock, vortex, etc. Then, the positions of crucial features are included in the adaptive dataset.
- **Step 2:** The flux evaluation is performed in the including cells in the adaptive dataset with conventional methods.
- **Step 3:** In the excluded cells in the adaptive dataset, the residual is computed from the results of Step 2 by residual interpolation.
- **Step 4:** The time integration is carried out. Then, restriction method is applied to the solution; negligible flow variations are restricted and the convergence of the solution is enhanced.

Figure 1: Overall procedure of multi-resolution analysis algorithm



(a) Adaptive dataset of 1 level resolution (b) Pressure coefficienct distribution at 50% of span

Figure 2: Adaptive dataset and pressure distribution on ONERA-M6

Table 1: Results of efficiency improvements and L_2 error

		Iteration	SPR Pt.	CPU Time	Time Ratio	L_2 Error
Reference Solver		10764	·	29329.3	·	·
Multi-resolution	1 Level	3206	64.61%	9192.5	3.19	2.51×10^{-7}
	2 Level	3267	64.57%	9438.4	3.11	3.14×10^{-7}
	3 Level	3205	64.74%	9251.8	3.17	3.49×10^{-7}

3 Conclusion

In this study, multi-resolution method for three dimensional Euler equations is proposed for enhancement of the computational efficiency. The algorithm is applied to flow computation of the field around ONERA-M6 wing in transonic regime. The numerical exercise shows that the algorithm speeds up the simulation about 3 times faster than that of a conventional CFD solver, while preserving the numerical accuracy of the original solution.

References

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