Accurate Sharp Interface Scheme for Multimaterials

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Abstract: We present a method to capture the evolution of a contact discontinuity separating two different materials. A locally non-conservative scheme allows an accurate and stable simulation while the interface is kept sharp. Numerical illustrations include impact problems involving compressible elastic media surrounded by air.

Keywords: Multimaterial scheme, Sharp interface, Second order.

1 Introduction

Physical and engineering problems that involve several materials are ubiquitous in nature and in applications: multi-phase flows, fluid-structure interaction, particle flows, to cite just a few examples. The main contributions in the direction of simulating these phenomena go back to [1] and [2] for the model. The idea is to model the eulerian stress tensor through a constitutive law reproducing the mechanical characteristics of the medium under consideration. A remarkable example based on this approach is presented in [3]. However, the numerical scheme presented in that paper is relatively complicated and has the disadvantage that the interface is diffused over a certain number of grid points. In this paper we propose a simple second-order accurate method to recover a sharp interface description keeping the solution stable and non-oscillating.

2 Problem Statement

The conservative form of elastic media equations in the eulerian framework are

$$\begin{cases} \rho_t + \mathrm{div}_x(\rho u) = 0 \\ (\rho u)_t + \mathrm{div}_x(\rho u \otimes u - \sigma) = 0 \\ (\rho e)_t + \mathrm{div}_x(\rho e u - \sigma^T u) = 0 \\ (\nabla Y)_t + \nabla_x(u \cdot \nabla_x Y) = 0 \end{cases} \tag{1}$$

The unknowns are the backward characteristics of the problem $Y(x,t)$, the velocity $u(x,t)$, the total energy per unit mass $e(x,t)$ and the density $\rho(x,t)$. Here $\sigma(x,t)$ is the Cauchy stress tensor in the physical domain. To close the system, a constitutive law which connects σ to the unknowns is given:

$$\varepsilon = e - \frac{1}{2}|u|^2 = \frac{\exp\left(\frac{s}{c_v}\right)\rho^{\gamma-1}}{\gamma - 1} + \frac{p_\infty}{\rho} + \frac{\chi}{\rho_0}(\mathrm{Tr}(\overline{B}) - 2) \tag{2}$$

where $s(x,t)$ is the entropy, $\overline{B}(\nabla Y)$ is the modified left Cauchy-Green tensor and $c_v, \gamma, p_\infty, \chi$ are constants that characterize a given material. The two first terms represent a stiffened gas and the third one accounts for Neo Hookean elastic solid. The stress tensor is then derived from this constitutive law. Our multimaterial scheme is based a directional splitting. The fluxes are computed by an HLLC approximate Riemann solver on a fixed cartesian mesh. The interface is kept sharp by using a non-conservative numerical flux for the computational cells that are crossed by the contact discontinuity. See [4] for the detailed multimaterial model (first-order scheme). The second-order scheme for rigid solids is described in [5].

3 Conclusion and Future Work

In Fig. 1 on the left we present an application of the multimaterial scheme with first-order accuracy in space. On the right we show results for the second-order scheme, but on a rigid body. Application of the second-order scheme to the multimaterial model is ongoing.

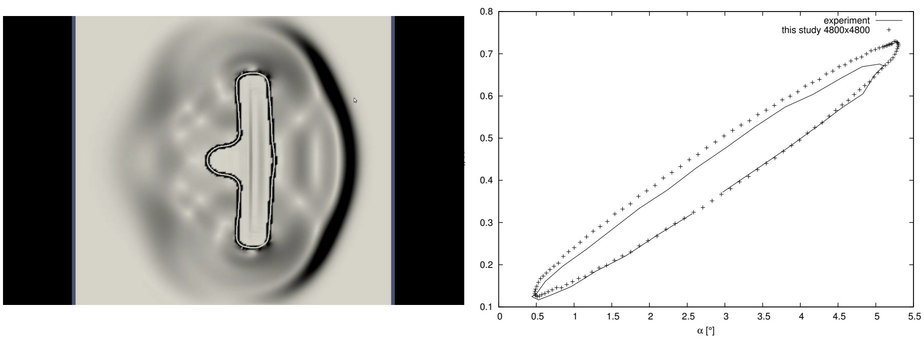

Figure 1: Left: Schlieren image of the density after the impact of a copper projectile surrounded by air. Right: application to a rigid oscillating solid airfoil; comparison of the normal force coefficient with experimental data from AGARD-R702.

References

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