Simulations of Compressible Rayleigh-Taylor Instability Using the Adaptive Wavelet Collocation Method

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Abstract: Numerical simulations of the compressible Rayleigh-Taylor instability are performed on an adaptive mesh using the Adaptive Wavelet Collocation Method (AWCM). Due to the physics-based adaptivity and direct error control of the method, AWCM is ideal for resolving the wide range of scales present in the development of the instability. The problem is initialized consistent with the solutions from linear stability theory, with two diffusively mixed, stratified fluids of differing molar masses as the background state. Of interest are the compressibility effects on the departure time from the linear growth, the onset of strong non-linear interactions, and the late-time behavior of the fluid structures. The late time bubble and spike velocities are computed and compared to those obtained in the incompressible case.

Keywords: Rayleigh-Taylor Instability, Adaptive Wavelet Collocation Method, Compressibility Effects.

1 Introduction

The use of a wavelet-based adaptive method for the simulation of complex fluid systems permits efficient use of computational resources, since high resolution simulations are performed only where small structures are present in the flow. Representation of the flow using wavelets allows the grid to dynamically adapt to the structures in the flow as they evolve in time while maintaining a direct control of the error [1]. The extension of AWCM to simulations of Rayleigh-Taylor instability is promising due to the localized nature of the system. The late-time behavior of the instability in the presence of compressibility and variable density effects is not currently fully understood [2, 3]. In order to capture the late time behavior, the simulations need to be performed in long vertical domains. The utilization of AWCM for simulations on such domains minimizes the computational effort, since Rayleigh-Taylor instability remains a spatially localized phenomenon near the interface well into the turbulent stage.

2 Problem Statement

In order to test the applicability of AWCM for direct numerical simulations of Rayleigh-Taylor instability, a two-dimensional, single-mode system is studied. Linear stability theory offers an

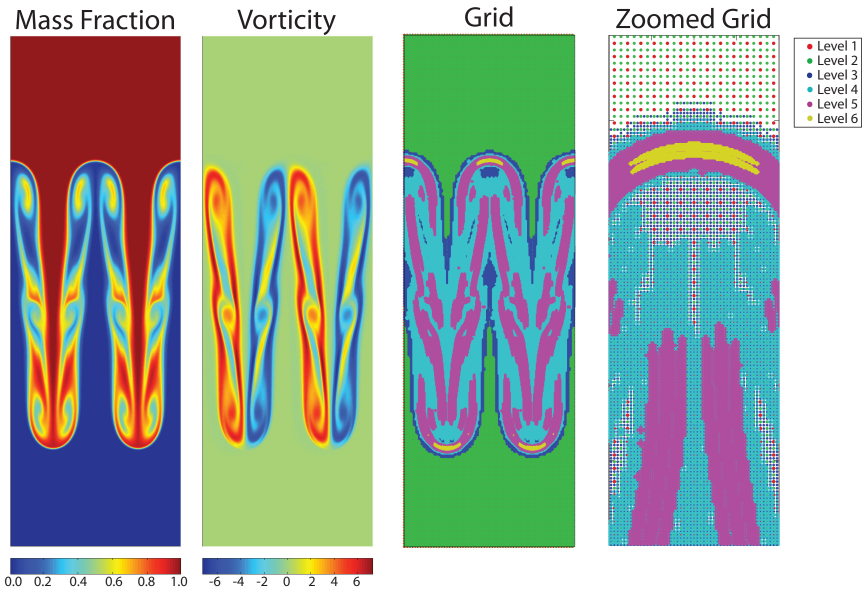

Figure 1: Mass fraction, vorticity, and the associated adaptive grid for the late-time instability growth.

early time solution to the instability growth, which is used to verify the accuracy of the method [4]. Of further interest is achieving the terminal bubble velocity [5], observing a reacceleration region [6], and finally entering a chaotic mixing phase. The adaptive grid for a late-time Rayleigh-Taylor instability simulation is shown in Figure 1. The effective global resolution is 10241×1024, yet only 3.6% of the points are used (380,166 points, 96.4% compression). The case presented in the figure is nearly-incompressible with small variable density effects.

The Rayleigh-Taylor instability develops because the top fluid molar mass is greater than that for the lower fluid, that is $W_1 > W_2$. Variable density effects are investigated by varying the Atwood number,

$$A = \frac{W_1 - W_2}{W_1 + W_2}. \tag{1}$$

In order to investigate compressibility effects, a distinction is made between fluid compressibility characterized by the values of the ratios of the specific heats, γ_1 and γ_2, and compressibility effects in response to the thermodynamic state of the system, characterized by a Mach number defining the size of a characteristic velocity relative to the speed of sound [4]. The definition is:

$$M = \sqrt{\frac{\rho_i g \lambda}{P_i}}, \tag{2}$$

where ρ_i and P_i are the interfacial density and pressure, respectively. For the thermal equilibrium case, M also determines the vertical variations of the equilibrium density and pressure profiles, and can thus also be regarded as a stratification parameter [2].

3 Conclusion and Future Work

The use of AWCM for direct numerical simulations of Rayleigh-Taylor systems is promising, since the spatial localization of the mixing layer leads to significant compression in the number of points necessary, while maintaining a high effective resolution and an explicit error control.

The numerical tests show that the method successfully captures the linear regime, bubble and spike formations, and late-time flow characteristics for the single-mode perturbation case. In order to observe the variable density and compressibility effects on the late time development of Rayleigh-Taylor instability, simulations at higher Atwood and Mach numbers are under way.

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