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Parallel Output-Adaptive Solution Strategies for Unsteady Aerodynamics on Deformable Domains

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Abstract: We present an output-based adaptation strategy for high-order discretizations on deformable domains. The equations are solved with a discontinuous Galerkin discretization using an arbitrary Lagrangian-Eulerian approach. Discrete unsteady adjoint solutions, derived for both the state and the geometric conservation law, provide scalar output error estimates and drive adaptive refinement of the space-time mesh. Spatial adaptation is performed using dynamic order refinement on a fixed tessellation of the domain. Temporal refinement consists of time slab resizing. In addition, a dynamic-repartitioning load balancing strategy is used for parallel computations. Results for the compressible Navier-Stokes simulations demonstrate accuracy of the error estimates and efficiency of the proposed output-based adaptation approach.

Keywords: Mesh Motion, Geometric Conservation Law, Discontinuous Galerkin, Output Adaptation, Unsteady Adjoint

1 Introduction

As computational fluid dynamics simulations become more complex, estimates of discretization error are of increasing interest for improving both robustness and accuracy. Output-error estimates are especially suited for this task as they provide numerical error bars on quantities of engineering interest. Furthermore, these estimates can be localized for adapting the mesh to improve output accuracy.

This work considers combined temporal and spatial refinement for unsteady simulations on deformable domains. Such simulations have far-reaching applications, from bio-inspired flight to aircraft maneuver and flutter analysis. The runs are generally computationally intensive and the resulting solutions are often rich in features. We show that for these problems output error estimation and adaptation can have a significant impact on robustness. The present research is a continuation of previous work in unsteady output-based adaptation on static meshes [1]. The discretization and error estimation extend naturally to the geometric conservation law employed on deformable domains, and the required modifications are discussed.

2 Discretization, Error Estimation, and Adaptation

We solve a system of PDEs on deformable domains by mapping the equations to a static reference domain using an arbitrary Lagrangian-Eulerian (ALE) approach [2], as illustrated in Figure 1. Due to the nonlinear and non-polynomial nature of general mappings, a constant state in the physical domain will generally not be representable using a standard polynomial basis in the reference domain. We address this problem with a geometric conservation law (GCL) as described in [2].

We employ an output-adjoint driven unsteady error estimation and adaptation strategy, as described in [1]. Specific to the case of mesh motion is the discretization of the adjoint equation associated with the GCL and the effect of this adjoint on error estimates. Adaptation is performed after localizing the adjoint-weighted residual error estimate to individual space-time elements, and after estimating the output error anisotropy by projecting the adjoint to solution spaces semi-refined in space or time.

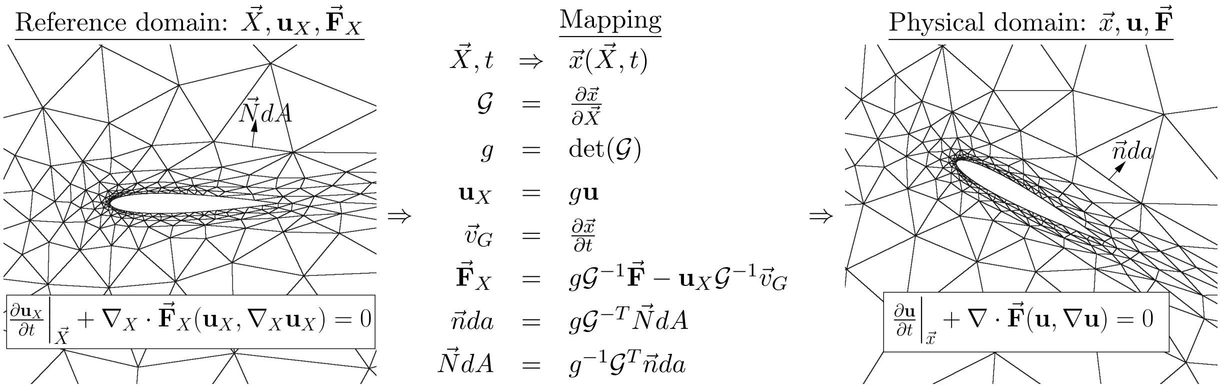
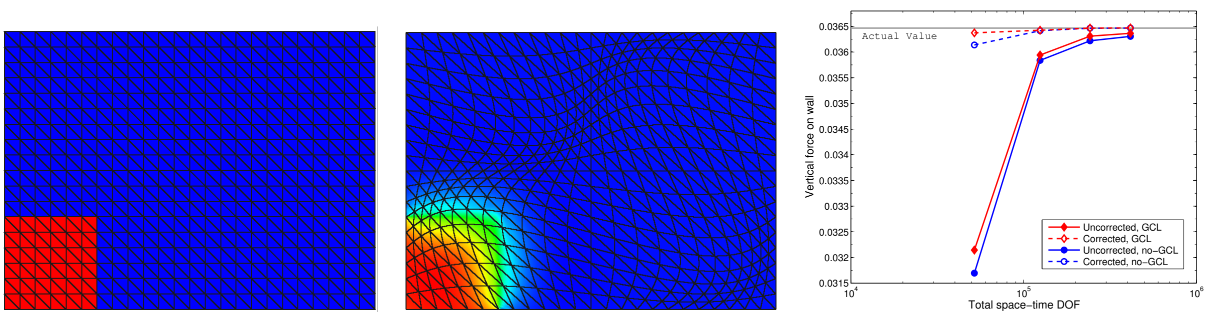


Figure 1: Summary of the mapping between reference and physical spaces.

3 Sample result

The result of a sample verification study for a problem that does not in fact require mesh motion is illustrated in Figure 2 and Table 1. In this case an initial square density perturbation is allowed to evolve for a short period of time, after which the vertical force on the left wall is measured. The system is governed by the compressible Navier-Stokes equations. The output convergence plot demonstrates the accuracy of the error estimates and the relatively small effect of the geometric conservation law on the output of interest.



(a) Density perturbation, $t = 0$ (b) Density, $t = 2$ (c) Output convergence

Figure 2: Initial- and final-time meshes and densities, and convergence of a final-time force output.

	GCL			No GCL			No Motion		
Run	δJ_{est}	ΔJ_{act}	% Error	δJ_{est}	ΔJ_{act}	% Error	δJ_{est}	ΔJ_{act}	% Error
$p = 1$	-4.170e-3	-4.020e-3	**3.73**	-4.443e-3	-4.489e-3	**1.03**	-1.687e-4	-2.882e-4	**41.44**
$p = 2$	-4.746e-4	-5.080e-4	**6.58**	-5.782e-4	-6.115e-4	**5.46**	-5.922e-5	-7.026e-5	**15.70**
$p = 3$	-1.551e-4	-1.543e-4	**0.49**	-2.448e-4	-2.438e-4	**0.40**	-1.610e-5	-1.555e-5	**3.51**
$p = 4$	-1.022e-4	-1.008e-4	**1.40**	-1.626e-4	-1.608e-4	**1.12**	-7.308e-6	-7.015e-6	**4.18**

Table 1: Relative accuracy of error estimates for motion and no-motion cases at different orders p.

References

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[2] P.-O. Persson, J. Bonet, and J. Peraire. Discontinuous Galerkin solution of the Navier-Stokes equations on deformable domains. *Computer Methods in Applied Mechanical Engineering*, 198:1585–1595, 2009.