

# High-Fidelity Flapping-Wing Aerodynamics Simulations with a Dynamic Unstructured Grid based Spectral Difference Method

Meilin Yu<sup>1</sup>, Z. J. Wang<sup>2</sup> and Hui Hu<sup>3</sup>

Department of Aerospace Engineering  
Iowa State University, Ames, IA 50011

**Abstract:** A dynamic unstructured grid based high-order spectral difference (SD) method has been developed to solve the three dimensional compressible Navier-Stokes (N-S) equations. This solver is adopted to simulate the flapping-wing aerodynamics at low Reynolds numbers. Both two dimensional and three dimensional bio-inspired flow problems have been studied and the efficiency and robustness of the solver have been demonstrated for such vortex-dominated flows. A two dimensional asymmetric wake phenomenon behind a pitching airfoil and the three dimensional flow fields around several flapping wings of different planforms have been presented in the paper.

*Keywords:* Spectral Difference, Navier-Stokes Equations, Flapping-Wing Aerodynamics, Vortex Dominated Flow.

## 1 Introduction

High-order computational fluid dynamics (CFD) methods have attracted a surge of research activities in recent years due to their efficiency and accuracy for vortex dominated flows. A review on the unstructured grid based high-order methods for the Euler and Navier-Stokes equations can be found in [2]. The spectral difference (SD) method [1] is a recently developed unstructured grid based high-order method to solve hyperbolic conservation laws. In [3], the SD method has been successfully extended for the deformable dynamic grid and demonstrated its capability to handle complex vortex dominated bio-inspired flow.

## 2 Problem Statements

The governing equation for the fluid is the unsteady compressible Navier-Stokes equations in conservation form, which reads,

$$\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} + \frac{\partial H}{\partial z} = 0. \quad (1)$$

Herein,  $Q = (\rho, \rho u, \rho v, \rho w, E)^T$  are the conservative variables,  $\rho$  is the fluid density,  $u, v$  and  $w$  are the Cartesian velocity components, and  $E$  is the total initial energy.  $F, G, H$  are the total fluxes including both the inviscid and viscous flux vectors, i.e.,  $F = F^i - F^v$ ,  $G = G^i - G^v$  and  $H = H^i - H^v$ .

To achieve an efficient implementation with the SD method, a time-dependent coordinate transformation from the physical domain  $(t, x, y, z)$  to the computational domain  $(\tau, \xi, \eta, \zeta)$  is applied on Eq. (1), which is

$$\frac{\partial \tilde{Q}}{\partial \tau} + \frac{\partial \tilde{F}}{\partial \xi} + \frac{\partial \tilde{G}}{\partial \eta} + \frac{\partial \tilde{H}}{\partial \zeta} = 0. \quad (2)$$

Detailed information on the construction with the SD method can be found in [3].

Both 2D and 3D bio-inspired flows over flapping wings are studied in the following paper. Since the characteristic flow speed in flapping wing aerodynamics is relatively small, a low Mach number preconditioning has been applied to the compressible solver to handle low Mach number flow. The performance of the developed solver for low Mach number flow is tested at first for a steady flow over a NACA0012 airfoil at  $Re = 1,200$ ,  $Ma_\infty = 0.05$  and zero angle of attack (AoA) with a 3<sup>rd</sup> order accurate scheme and an implicit LU-SGS time integration on a coarse mesh. As shown in Fig. 1 the solver works efficiently at low Mach number as no oscillation of the flow field is observed. Two dimensional simulations of a pitching NACA0012 airfoil with the pivot at the quarter-chord location and the

<sup>1</sup> Graduate Student, Department of Aerospace Engineering. Email: [mlyu@iastate.edu](mailto:mlyu@iastate.edu)

<sup>2</sup> Wilson Professor of Engineering, Department of Aerospace Engineering. Email: [zjw@iastate.edu](mailto:zjw@iastate.edu)

<sup>3</sup> Associate Professor, Department of Aerospace Engineering. Email: [huhui@iastate.edu](mailto:huhui@iastate.edu)

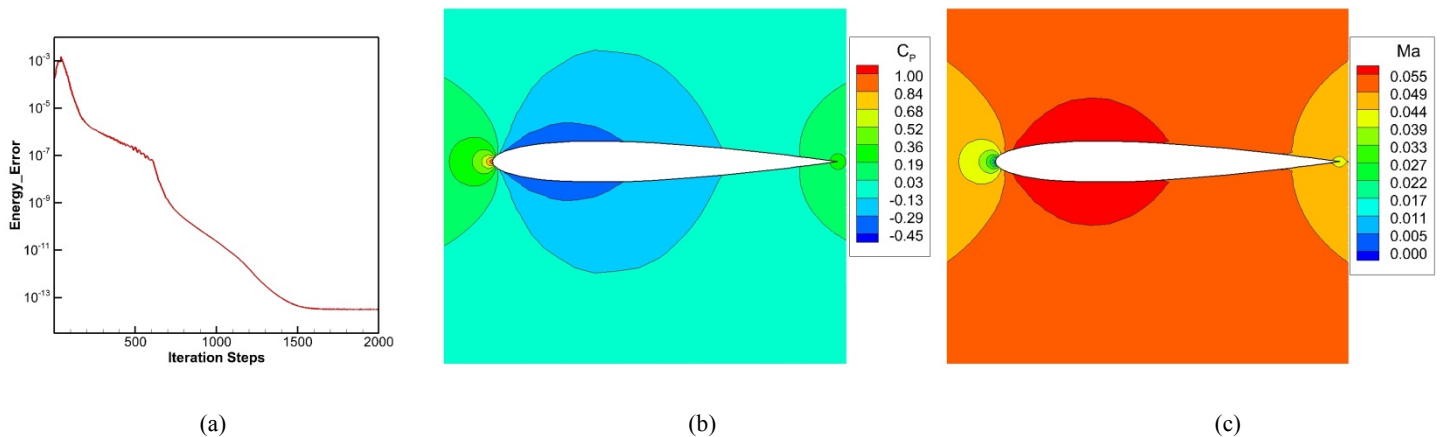
comparisons with the experimental results will be displayed in the final papers. Three wing planforms, namely rectangular, elliptic and bio-inspired (hawk moth) flapping wings are used for the 3D simulations. Some preliminary results around the bio-inspired wing are displayed in Fig. 2. More simulation results will be shown in the final paper.

### 3 Conclusion and future work

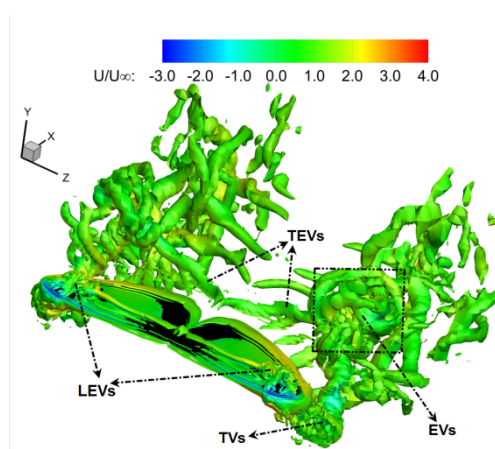
A dynamic unstructured grid based high-order spectral difference compressible Navier-Stokes solver is used in the present study to perform high-fidelity simulations on both the 2D and 3D bio-inspired flows. The solver works efficiently for the bio-inspired flows and can well capture complex elaborate vortex structures. More simulation results will be shown in the final paper.

### References

- [1] Liu, Y., Vinokur, M. and Wang, Z. J., Discontinuous spectral difference method for conservation laws on unstructured grids, *J. Comput. Phys.* (2006), vol. 216, pp. 780–801.
- [2] Wang, Z. J., High-order methods for the Euler and Navier Stokes equations on unstructured grids, *Progress in Aerospace Science* (2007), vol. 43, pp. 1-41.
- [3] Yu, M. L., Wang, Z. J., and Hu, H., A high-order spectral difference method for unstructured dynamic grids, *Computer & Fluids*, Vol.48, No. 1, 2011, pp.84-97.



**Fig. 1 .** (a) Convergence history of the energy error for the steady solution of the flow over a stationary NACA0012 airfoil with implicit (LU-SGS) time integration at  $Ma_\infty = 0.05, Re = 1,200$ ; (b) pressure coefficient contours for the converged steady flow; (c) Mach number contours for the converged steady flow.



**Fig. 2 .** Vorticity field visualizations of an flapping bio-inspired wing at Reynolds number  $Re = 1,200$ , Strouhal number  $Str = 0.38$ , and reduced frequency  $k = 3.5$ . The vortex structures are indicated by the Q-criteria and colored by the stream-wise velocity.