High-Order Discontinuous Galerkin Methods for Turbulent High-lift Flows

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Abstract: In this work a robust discontinuous Galerkin (DG) solver for turbulent high-lift aerodynamic flows using the turbulence model of Spalart and Allmaras (SA) is developed. The application of DG discretizations to turbulent RANS flows is one of the most pressing issues facing high-order methods on unstructured grids. The issue is the result of non-smooth behavior of the turbulence model equation, which often causes solver failure for high-order discretizations. Herein a modification to the turbulence model of Spalart and Allmaras is implemented within a DG-based unstructured CFD solver for the RANS equations and utilized to compute high-lift aerodynamic flows.

Keywords: High-Order Methods, Computational Fluid Dynamics, Turbulence Modeling.

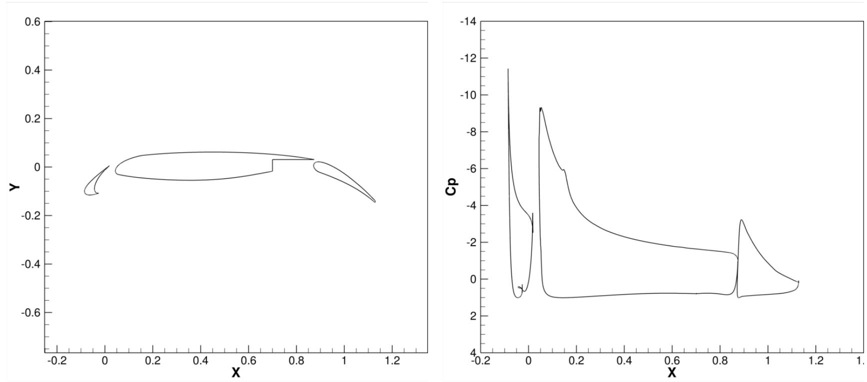
1 Introduction

Recent research[4] into the robust application of discontinuous Galerkin (DG) methods to the Reynolds Averaged Navier-Stokes (RANS) equations has proven difficult, due to non-smooth solution behavior. While the solutions of the mean flow quantities are smooth the solution of the Spalart Allmaras (SA) turbulence model equation[1] are non-smooth. Recent work[2] has shown that the non-smooth behavior of the turbulence model equation often causes solver failure, especially for high-lift applications. Herein a modified version of the SA turbulence model equation given by reference[3], which alleviates the difficulties encountered with the SA turbulence model non-smooth behavior, is implemented in a high-order DG solver[4].

While there are a few examples of successful high-order DG RANS solutions[2], which do not employ modifications to the turbulence model equation, these solutions are relegated to relatively benign flows at low, Mach numbers, Reynolds numbers and angles of attack. Furthermore, obtaining high-order solutions for the original form of the SA model is far from robust. This work will demonstrate that the present DG solver, using high-order discretizations of the RANS and modified SA turbulence model[3] equations, is capable of computing high-lift flows without difficulty.

2 Preliminary Results

Demonstrating the robustness of the present DG solver employing the modified turbulence model of reference[3], requires computing a challenging test case. For this purpose, the flow over a three-element high lift configuration at the flow conditions $M_\infty = .2$, $\alpha = 16^o$, and $Re = 9,000,000$ is considered. The computed surface pressure coefficient obtained using a DG solver with a discretization order of $p = 3$ (i.e. 4^{th} order accurate) is depicted in Figure 1(b).



(a) Three-element airfoil configuration

(b) Surface pressure using $p = 3$

Figure 1: Airfoil configuration and computed surface pressure coefficient using $p = 3$ for flow over a high-lift three-element airfoil configuration at $M_\infty = .2$, $\alpha = 16^o$, and $Re = 9,000,000$.

3 Future Work

This work will demonstrate that the difficulties related to computing turbulent flows with high-order DG methods, are due entirely to the non-smooth behavior of turbulence model equation. If the non-smooth behavior is suitably addressed, solvers based on high-order DG discretizations are capable of solving practical aerodynamic flows. Future work will present a validation of the present DG solver for high angle of attack airfoils as well as high-lift configurations. The robustness of this approach will be demonstrated by considering a wide range of aerodynamic applications.

References

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